Price Smoothing under Capacity Constraints*

KON S. LAI
California State University
Los Angeles, California

I. Introduction

This paper examines monopolistic pricing decisions under production and storage capacity constraints, which firms may well encounter in practice, but which have generally been ignored in earlier price-inventory analyses.[1] If these capacity constraints are present, they evidently influence firms' pricing decisions. These constraints should therefore be formally modelled for a more complete theory of price behavior.

The paper addresses the issue of whether or not inventories will lead to an asymmetry in price behavior. Amihud and Mendelson [2] and Reagan [15] have noted that inventory adjustments moderate downward price adjustments in periods of low demand, but do not moderate upward price adjustments in periods of high demand when stockouts occur. These analyses are in contrast to Blinder [4], wherein a possible symmetric role of inventories and unfilled orders in dampening price adjustments is considered. Blinder argues that stockouts may not be empirically significant. Abel [1] observes, however, that casual empiricism may lead one to think that stockouts are rather uncommon, but the fact that an individual is only rarely rationed offers no evidence that, from the firm's point of view, stockouts are unusual. In addition, a recent analysis by Kahn [7] suggests a role of the stockout-avoidance motive in understanding the "excess" volatility of production documented in empirical work, e.g., Blinder [5].

In this paper it is shown that the Amihud-Mendelson-Reagan's result may derive from prospective stockouts rather than realized stockouts, as in their analyses. When price behavior can depend upon the prospective state of inventories, the fact that stockouts are not often observed will not necessarily imply that stockouts have little bearing on pricing decisions.

It is shown, however, that the result on an asymmetry in price behavior depends crucially on an implicit assumption that the firm faces no effective storage capacity constraint. Inventories cannot accumulate without limit, and they are always competing for given storage spaces. In periods of low demand the firm's inventories accumulate through time and when the storage capacity constraint is about to be binding, price becomes more responsive. This fits in with casual observations that prices do tend to fall fairly sharply when firms, which face huge inventories, try to liquidate their stocks and increase turnovers in response to weaker demand. Hence, in the

*The author would like to thank Peter Pauly, Richard Roseman, and the referee for useful suggestions. The usual caveat applies.

1. Recent price-inventory analyses include Abel [1], Amihud and Mendelson [2], Ashley and Orr [3], Blinder [4], Kahn [7], Maccini [9], Reagan [15], and Zabel [18].

presence of storage capacity constraints, the asymmetry in price behavior noted in earlier analyses will have a symmetric counterpart.

The paper is organized as follows. Section II outlines the model with storage and production capacity constraints. Section III examines the effects of the capacity constraints on price behavior. The corresponding implications for the aggregate supply curve are discussed in section IV. Concluding remarks are contained in section V.

II. The Model

Following Ashley and Orr [3], this analysis focuses exclusively on the production smoothing motive for holding inventories, and the uncertainty buffering motive is not considered. The firm's problem examined involves the determination of optimal pricing and production policies in face of foreseeable (probably nonstationary) fluctuations in demand. Ashley and Orr note that such price smoothing model typically applies to seasonal demand shifts.

The analysis, like that of Ashley and Orr, is undertaken in terms of a deterministic framework for the sake of mathematical tractability, but still enables us to establish insights into the effects of capacity constraints on firms' decisions. Maccini [9] observes that intertemporal models appear in general to be mathematically intractable when uncertainty can enter in a complex way. Indeed, closed form solutions are not generally available except for a specific class of models, namely linear-quadratic models with additive uncertainty for which we can appeal to the existence of certainty equivalence, as in Blinder [4; 5]. Schutte [16] notes, however, that when a nonnegativity constraint on inventories is imposed, Blinder's model will lose its certainty equivalence property, required to derive a closed form solution of the model.

The introduction of boundary constraints can further complicate the price smoothing analysis in another way that deserves discussion. When demand is stochastic, the inventory level, which is a function of the initial inventory and cumulative demand, will also be stochastic and become negative at some points in time. As a result, the use of a deterministic nonnegativity constraint on inventories, often adopted in earlier work, will not be appropriate if not inconsistent with the stochastic framework. Instead of a deterministic constraint, a chance constraint on inventories can be used in the stochastic case. The chance constraint specifies the inventory level to be nonnegative with some desired level of probability chosen by the firm. Parlar [14], for example, shows that if the cumulative demand follows a compound Poisson process, the problem of production smoothing with chance constraints can be transformed into a deterministic optimal control problem with inequality constraints on control and state variables. Parlar's result provides a rationalization for the deterministic approach, applied in this paper.

Consider that a monopolist, facing a shifting demand curve, makes plans for price p_t and production x_t at the beginning of period zero and with initial inventory v_0. The demand schedule is assumed to be given by

$$y_t = Q(p_t; a_t), \tag{1}$$

where a_t denotes the anticipated demand shift in period t, and $dy_t/da_t > 0$, $dy_t/dp_t < 0$, and $2(dy_t/dp_t)^2/y_t > d^2y_t/dp_t^2$. The last condition says that the marginal revenue $MR(p_t) = p_t + y_t/(dy_t/dp_t)$ is an increasing function of price. Denote by $C(x_t)$ and $H(v_t)$ the production and inventory cost functions, that are assumed to be increasing and convex. The dynamics of inventories are governed by

$$v_{t+1} = v_t + x_t - y_t. \tag{2}$$

The nonnegativity and capacity constraints on inventories are described by

$$v_t \geq 0, \quad v_t \leq \bar{v}, \tag{3}$$

where $\bar{v} > 0$. Following Mills [12], we are interested in the case where the optimal production rate is positive and the production nonnegativity constraint is ignored. The capacity constraint on production is represented by

$$0 < x_t \leq \bar{x}. \tag{4}$$

The assumption of rigid capacities is a convenient simplification that will serve our turn; we may consider them as boundaries beyond which marginal costs rise exceedingly steeply. By imposing fixed capacities and using inequality constraints, this approach provides a tractable framework that captures the effective supply of output and, as will be shown, leads to results more general than that obtained from existing inventory models.[2]

The firm is considered to maximize the sum of discounted expected profits. Let r denote the rate of interest. The Lagrangian of the firm's maximization problem, subject to (2), (3), and (4), over a planning horizon T is given by

$$\sum_{t=0}^{T} (1+r)^{-t}\{p_t Q(p_t; a_t) - C(x_t) - H(v_t)\} + \phi_t(-v_{t+1} + v_t + x_t - y_t)$$
$$+ \theta_t v_t + \xi_t(\bar{v} - v_t) + \varphi_t(\bar{x} - x_t), \tag{5}$$

where θ_t, ξ_t, and φ_t are the corresponding Lagrange multipliers, and ϕ_t is the adjoint variable for (2). If the revised plans always look a fixed number of periods into the future, T will be a constant. If the firm plans toward an unchanging horizon, however, T will be a periodic function of time.[3]

This paper will not deal with the question of the determination of the length of the planning horizon. Lee and Orr [8] shows that any planning horizon depends critically on the existence of "bottleneck" conditions arising either from a storage capacity constraint or an inventory nonnegativity constraint. The latter is considered in, e.g., Ashley and Orr [3] and Modigliani and Hohn [13]. When production and storage capacity constraints are considered, as in the present paper, Thompson, Sethi, and Teng [17] show that an optimal planning horizon exists.

In view of (5) the necessary conditions for optimality are given by

2. At the theoretical level, an alternative to assuming capacity constraints is to assume convex cost functions, as considered in Blinder [4]. Blinder shows that price behavior depends upon the curvature of the production and inventory cost functions. The quadratic cost functions are, nonetheless, not sufficient to produce the price regimes results derived later in this paper. In order to produce these results, we need to modify the quadratic specification of the model and manipulate the shapes of the production and inventory cost functions. Unfortunately, in that case a closed form solution to Blinder's model appears not possible.

3. Specifically, T takes the largest value at certain points in time at the beginning of a planning horizon. Thereafter, the terminal date of the horizon remains unchanged so that T gradually shrinks to a minimum, after which a new planning horizon is relevant again.

$$C'(x_t) = \phi_t - \varphi_t, \tag{6}$$

$$H'(v_t) = \phi_t - (1+r)\phi_{t-1} + \theta_t - \xi_t, \tag{7}$$

$$MR(p_t; a_t) = \phi_t, \tag{8}$$

and (2); the transversality condition is

$$\phi_T \geq 0, \quad \phi_T v_{T+1} = 0; \tag{9}$$

and the complementarity and nonnegativity conditions are

$$\theta_t v_t = 0, \qquad \theta_t \geq 0, \quad v_t \geq 0; \tag{10a}$$

$$\xi_t(\bar{v} - v_t) = 0, \qquad \xi_t \geq 0, \quad \bar{v} \geq v_t; \tag{10b}$$

$$\varphi_t(\bar{x} - x_t) = 0, \qquad \varphi_t \geq 0, \quad \bar{x} \geq x_t. \tag{10c}$$

Under the model assumptions the conditions are also sufficient for optimality [11].

Ashley and Orr show that within an optimal planning horizon, there is a tendency for price to rise through time: a downward demand shift can be accompanied by a higher price, but price will definitely rise with an upward demand shift. They observe that this price-rising tendency may be manifested as sticky pricing behavior. Ashley and Orr's result can be demonstrated here in a slightly more general form. Combining (6), (7), and (8) yields

$$MR(p_{t+1}; a_{t+1}) - MR(p_t; a_t) = r(C'(x_t) + \varphi_t) + H'(v_{t+1}) - \theta_{t+1} + \xi_{t+1}. \tag{11}$$

This condition subsumes the interperiod condition derived by Ashley and Orr (their equation (9)), wherein capacity constraints are not considered and production costs play no part. A positive inventory level in period t and inactive capacity constraints imply that φ_t, θ_{t+1}, and ξ_{t+1} all equal zero. Hence, in Ashley and Orr's case relationship (11) becomes

$$MR(p_{t+1}; a_{t+1}) - MR(p_t; a_t) = rC'(x_t) + H'(v_{t+1}) > 0, \tag{12}$$

since $C'(x_t) > 0$ and $H'(v_{t+1}) > 0$. Since MR is an increasing function of the price, (12) implies that $p_{t+1} > p_t$ even when $a_{t+1} = a_t$, i.e., even when demand shifts are the same in the two periods. It follows that there is a tendency for the price to increase over the horizon. Note that the higher the interest rate or the marginal production and marginal inventory costs are, the greater will be the tendency for the price to rise within the horizon, and the stickier the price will be according to Ashley and Orr.[4]

The problem as posed generally above cannot yet be given a constructive explicit solution. When more definite results are sought, it seems necessary to make simplifying assumptions about the functions Q, H, and C. In view of various special forms treated in previous work, we consider a linear demand curve, quadratic production costs, and linear inventory costs:

$$y_t = a_t - 2bp_t, \quad b > 0; \tag{13}$$

$$C(x_t) = cx_t^2/2, \quad c > 0; \tag{14}$$

$$H(v_t) = hv_t, \quad h > 0. \tag{15}$$

4. This notion of price stickiness differs from that considered by, e.g., Blinder [4] and Reagan [15]. While Ashley and Orr are interested in the direction of price movements through time, Reagan and Blinder are concerned with the magnitude of price responsiveness to demand changes.

The addition of a linear term to $C(x_t)$ will not affect the following analysis. The model specification is similar to that in Ashley and Orr. While this is less general than that in Blinder [4], where quadratic inventory costs are considered, it is more general than that in Amihud and Mendelson [2] and Reagan [15], where linear production costs are considered. Corresponding to (6), (7), and (8), the set of optimality conditions is given by

$$cx_t = \phi_t - \varphi_t; \tag{16}$$

$$h = \phi_t - (1+r)\phi_{t-1} + \theta_t - \xi_t; \tag{17}$$

$$2p_t - a_t/2b = \phi_t. \tag{18}$$

Combining (2), (13), (16), and (18) gives

$$v_{t+1} = v_t + (1/c + b)\phi_t - \varphi_t/c - a_t/2. \tag{19}$$

Since the firm can revise its plans at the beginning of every period based on a new forecast of demand changes, we will be interested in the first-period optimal policy. Specifically, if we can obtain the solution value for ϕ_0, then we can solve explicitly for the optimal pricing decision for the current period. Substituting successively for v_t into (19) yields

$$v_{T+1} - v_0 = \sum_{t=0}^{T}\{(1/c + b)\phi_t - \varphi_t/c - a_t/2\}. \tag{20}$$

Likewise, from (17) we obtain

$$\phi_t = (1+r)^t\phi_0 + \sum_{j=1}^{t}\{(1+r)^{t-j}(h - \theta_j + \xi_j)\}. \tag{21}$$

Combining (20) and (21) then gives

$$\begin{aligned} v_{T+1} - v_0 =& (1/c + b)\phi_0/R(T) - \sum_{t=0}^{T}(a_t/2 + \varphi_t/c) \\ &+ \sum_{t=1}^{T}\sum_{j=1}^{t}\{(1+r)^{t-j}(h - \theta_j + \xi_j)\}, \end{aligned} \tag{22}$$

where $R(T) = 1/\sum_{t=0}^{T}(1+r)^t$ so that $dR/dr < 0$. For an optimal planning horizon, Ashley and Orr show that $v_{T+1} = 0$. This condition is also consistent with the transversality condition. It then follows from (22) that

$$\phi_0 = R(T)\{\overline{a}/2 + \sum_{t=1}^{T}[(\theta_t - \xi_t)/R(T-t)] + \sum_{t=0}^{T}\varphi_t/c - v_0\}/(1/c + b), \tag{23}$$

where $\overline{a} = \sum_{t=0}^{T} a_t$, which represents the sum of subsequent demand shifts.

The basic idea behind the solution procedure is straightforward. When there is no violation of the constraints, the solution to the unconstrained problem is optimal. When some violations exist, however, we adjust for jumps in the Lagrange variables at entering the boundaries of the constraints and move ϕ_t up and down in different periods such that it leads to an optimal feasible solution. Hence, except when all the constraints are not active, ϕ_0 is a function of $\overline{a}$, but also of φ_t and $\theta_t - \xi_t$. A general discussion of the behavior of the solution paths for optimal control problems with boundary constraints can be found in, e.g., McIntyre and Paiewonsky [10].

III. Different Regimes of Price Behavior

With the introduction of inequality constraints on inventories and production, the model can generate different regimes of price behavior according to the states of inventories and production capacity utilization. The regimes are demarcated according to the boundaries of the constraints in storage capacity, production capacity and stockouts. It will be shown that when the optimal plans contain a boundary segment of any of the constraints, the implied price behavior is rather different from that when the constraints are irrelevant. Mathematically, when the optimal paths of the control variable (production) and the state variable (inventories) enter into the boundary of a constraint in any period, its corresponding Lagrange variable jumps from a zero to a positive value, causing a jump in the shadow value of the current-period inventories ϕ_0. This jump behavior of ϕ_0 leads to different regimes of price behavior.

As noted earlier, Reagan [15] suggests an asymmetry in price behavior arising from the inventory nonnegativity constraint: price responds more strongly to demand changes when there is a stockout than when there is not. Reagan's proposition can be established here. To obtain the desired derivative of decision variables with respect to anticipated demand changes, we first obtain the total derivative of equation (23), given by

$$\begin{aligned} d\phi_0 = R(T)\{1/2 + \sum_{t=1}^{T} [\partial\theta_t/\partial\overline{a} - \partial\xi_t/\partial\overline{a})/R(T-t)] \\ + (\sum_{t=0}^{T} \partial\varphi_t/\partial\overline{a})/c\}d\overline{a}/(1/c + b). \end{aligned} \tag{24}$$

Note that we have set $dv_0 = 0$ and the effects of unanticipated demand changes through changes in inventories will not be considered, since anticipated and unanticipated demand changes are found to produce qualitatively similar results. Combining (18) and (24) yields[5]

$$\begin{aligned} dp_0/d\overline{a} = R(T)\{1/2 + \sum_{t=1}^{T} [(\partial\theta_t/\partial\overline{a} - \partial\xi_t/\partial\overline{a})/R(T-t)] \\ + (\sum_{t=0}^{T} \partial\varphi_t/\partial\overline{a})/c\}/[2(1/c + b)] + (da_0/d\overline{a})/4b. \end{aligned} \tag{25}$$

When capacity constraints are ignored, as in Reagan's analysis, (25) becomes

$$dp_0/d\overline{a} = R(T)\{1/2 + \sum_{t=1}^{T} (\partial\theta_t/\partial\overline{a})/R(T-t)\}/[2(1/c + b)] + (da_0/d\overline{a})/4b. \tag{26}$$

Consider the effects on price of demand shifts that cause a stockout at the end of the current period, i.e., $v_1 = 0$. The stockout implies that $\partial\theta_1/\partial\overline{a} > 0$. When no stockout occurs, however, $\partial\theta_1/\partial\overline{a} = 0$. It follows from (26) that price responds more strongly to demand shifts when a stockout occurs than when it does not, as shown in Reagan.

While the present model considers foreseeable demand changes, Reagan's model considers demand uncertainty in the way that the demand changes are identically and independently distributed over time with mean zero. Since the pricing and output decisions are assumed to be made after the realization of the demand shocks, Reagan's model in effect reduces to a certainty model, nonetheless. In addition, the deterministic setting of the present model is not so restrictive as it

5. If the optimal horizon is affected by the pattern of demand shifts, they will also affect the optimal pricing decision through changing T. Such indirect effects will not be considered in this paper.

may seem since a demand change not foreseen can affect decisions through changes in inventories and the firm can revise plans every period with updated information. Moreover, while expected demand changes are zero in Reagan's model, demand changes in the model here can take any pattern over time, including nonstationary demand changes.

We have derived Reagan's result not in a different framework only. While Reagan shows that price responsiveness depends on the realized state of inventories, a more general result can be established here:

PROPOSITION 1. *Price is more responsive to expected demand changes when a stockout has occurred or is expected to occur beyond the current period.*

Proof. It follows directly from (26) that changes in price will be larger when $\partial\theta_t/\partial\bar{a} > 0$ for some t than when $\partial\theta_t/\partial\bar{a} = 0$ for $1 \leq t \leq T$.

This indicates that price behavior depends not only on the realized state of inventories in the current period but also on the prospective state of inventories beyond the current period.

The basic point of Reagan's argument for an asymmetry in price behavior, as in Amihud and Mendelson [2], is to suggest that a shortage of stocks would be much more "alarming" than the existence of surplus stocks. The presence of some limit to storage capacity in practice, however, leads to an argument symmetric to Reagan's. In response to large upward demand shifts the firm draws down its inventories until a stockout is about to occur and price becomes more responsive to the demand changes. On the other hand, in response to large downward demand shifts the firm's inventories accumulate until the storage capacity limit is going to be binding and price becomes more responsive. It follows that the storage capacity constraint leads to price behavior which is more symmetric than that obtained without it.

The effects of the storage capacity constraint on price behavior can be summarized as follows:

PROPOSITION 2. *Price is more responsive to expected changes in demand when the storage capacity is about to be exhausted than when it is not.*

Proof. The proposition follows from (25) that the change in price is larger when $\partial\xi_t/\partial\bar{a} > 0$ for some t than when $\partial\xi_t/\partial\bar{a} = 0$ for $1 \leq t \leq T$.

The result is intuitive. Inventory adjustments moderate price responsiveness to demand changes. When desired inventory adjustments are restricted by the storage capacity constraint, however, the firm has to use larger price adjustments to accommodate the demand changes.

In parallel to the effects of stockouts on price behavior, which would not exist if a symmetric role of unfilled orders is allowed for, production capacity constraints have similar effects on pricing decisions as stockouts. As demand rises, the firm is induced to raise production until it produces at capacity and then price becomes responsive in accommodating demand changes. This is summarized in the next proposition.

PROPOSITION 3. *Price is more responsive to expected demand shifts when the production capacity limit is expected to be effective when it is not.*

Proof. The proposition follows from (25) that the change in price is larger when $\partial\varphi_t/\partial\bar{a} > 0$ for some t than when $\partial\varphi_t/\partial\bar{a} = 0$ for $1 \leq t \leq T$.

The result indicates that production smoothing attenuates downward pressure on price in periods of low demand, and that price is more responsive when the production capacity limit becomes effective and restricts the degree of production smoothing in periods of high demand.

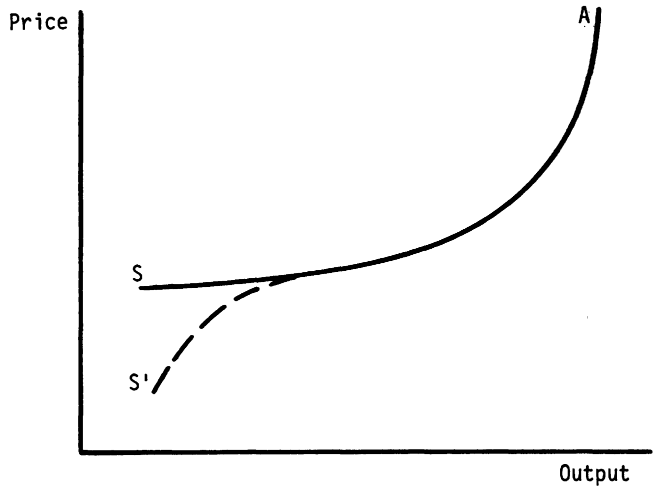

Figure 1. The Aggregate Supply Curve

In summary, effective capacity constraints on production and inventories as well as stockouts can lead to distinctly different price regimes. Under "normal" demand situations with production and storage at below capacity as well as no stockout, production and inventory adjustments moderate price fluctuations in response to changes in demand. In periods of high demand and when desirable production adjustments are restricted by the capacity limit or a stockout is about to occur, price becomes more responsive to demand changes. In periods of low demand, on the other hand, the firm's inventories accumulate through time. When the storage capacity limit is going to be binding, price becomes more responsive to demand changes.

IV. Implications for the Aggregate Supply Curve

A recent study by Evans [6] demonstrates the importance of production capacity constraints, referred as "bottlenecks," in determining the shape of the short-run aggregate supply curve and dynamics of output and inflation. Evans observes that modelling the effects of bottlenecks is essential for developing a microeconomic basis for macroeconomics. By explicitly modelling the production capacity constraint, a Keynesian demand-driven model can be extended to incorporate the effective supply of output. The resulting model displays both the Keynesian sticky price behavior and the classical non-sticky price behavior. Such approach is very much in the spirit of Keynes's analysis in Chapter 21 of *The General Theory*. Based on the point that bottlenecks facing individual firms will not be reached simultaneously, but successively as aggregate output increases, Evans is able to derive an upward sloping aggregate supply curve without assuming diminishing returns as in the conventional approach. Evans shows that, for a given distribution of bottlenecks, aggregation of individual supply curves over firms yields an upward sloping L-shaped aggregate supply curve, as indicated by curve AS in Figure 1.

The analysis here is similar to that of Evans in that the implications of bottlenecks for price behavior are explored. In contrast to Evans's one-period analysis, however, this analysis examines the firm's decision problem in a multiperiod setting and inventories with storage capacity constraints are introduced. It turns out that the existence of storage capacity constraints in addition to production capacity constraints implies a different shape of the short-run supply relationship from that derived by Evans. The point is that in periods of very low demand, price may become more responsive to demand changes as more and more firms face "excessive" inventories. It follows that the short-run supply curve should be relatively steep in the very low demand region, resulting in an S-shaped curve as indicated by curve AS' in Figure 1.

Note that the slope of the aggregate supply curve changes only gradually, and not sharply, in the high-demand and low-demand regions. This reflects that the bottlenecks will not be reached simultaneously at all firms, but successively as output changes. Evans suggests that the slope of the supply curve will vary with different distributions of the bottlenecks among firms.

V. Conclusions

A multiperiod pricing and production scheduling problem for a single storable commodity has been examined. The analysis differs from previous analyses in explicitly introducing production and storage capacity constraints, which firms may well encounter in practice. By formally incorporating the capacity constraints, the analysis provides a microeconomic basis for price behavior in face of possible bottlenecks. Moreover, the results in this paper corroborate some of the earlier results in Blinder [4], Amihud and Mendelson [2], Ashley and Orr [3], Evans [6], and Reagan [15] but modify them in several interesting aspects. They are summarized as follows:

(1) Prospective stockouts, not just realized stockouts, will tend to increase price responsiveness. It follows that the fact that stockouts are not often observed does not necessarily suggest that the stockout-avoidance motive has little bearing on the firm's pricing decision.

(2) Inventories do not generally reduce price responsiveness in all states of demand or in any specific direction only. It is demonstrated that inventory holdings can lead to different regimes of, rather than an asymmetry in, price behavior according to the state of demand. Specifically, an asymmetry arising from an inventory nonnegativity constraint will have a symmetric counterpart when a storage capacity constraint is also considered.

(3) In macroeconomic terms, the analysis suggests that there is a potentially wide range of states of inventory holdings and production capacity utilization inside which prices are relatively "sticky," but outside which prices become responsive to demand fluctuations. The existence of different price regimes resembles the traditional classification of the Keynesian regime and the classical regime in macroeconomic theories. This analysis suggests, however, a different shape of the aggregate supply relationship from the conventional assumption in macroeconomics. Specifically, the analysis suggests in the aggregate an S-shaped rather than L-shaped supply curve. Research work on the empirical relevance of storage capacity constraints which drive the result is certainly of interest.

References

1. Abel, Andrew B., "Inventories, Stock-Outs and Production Smoothing." *Review of Economic Studies*, April 1985, 283–93.

2. Amihud, Yakov and Haim Mendelson, "Price Smoothing and Inventory." *Review of Economic Studies*, January 1983, 87–98.

3. Ashley, Richard A. and Daniel Orr, "Further Results on Inventories and Price Stickiness." *American Economic Review*, December 1985, 964–75.

4. Blinder, Alan S., "Inventories and Sticky Prices: More on the Microfoundation of Macroeconomics." *American Economic Review*, June 1982, 334–48.

5. ———, "Can the Production Smoothing Model of Inventory Behavior be Saved?" *Quarterly Journal of Economics*, August 1986, 431–54.

6. Evans, George, "Bottlenecks and the Phillips Curve: A Disaggregated Keynesian Model of Inflation, Output and Unemployment." *Economic Journal*, June 1985, 345–57.

7. Kahn, James A., "Inventories and the Volatility of Production." *American Economic Review*, September 1987, 667–79.

8. Lee, Dwight R. and Daniel Orr, "Further Results on Planning Horizons in the Production Smoothing Problem." *Management Science*, January 1977, 490–8.

Maccini, Louis J., "The Interrelationship between Price and Output Decisions and Investment Decisions." *Journal of Monetary Economics*, January 1984, 41–65.

10. McIntyre, John and Bernard Paiewonsky. "On Optimal Control with Bounded State Variables," in *Advances in Control Systems: Theory and Applications*, Vol. 5, edited by L. T. Leondes. New York: Academic Press, 1967.

Mangasarian, Olvi L., "Sufficient Conditions for the Optimal Control of Nonlinear Systems." *SIAM Journal on Control*, 1966, 139–52.

12. Mills, Edwin S. *Price, Output, and Inventory Policy*. New York: John Wiley, 1962.

Modigliani, Franco and Franz E. Hohn, "Production Planning Over Time and the Nature of the Expectation and Planning Horizon." *Econometrica*, January 1955, 46–66.

Parlar, Mahmut, "A Stochastic Production Planning Model with a Dynamic Chance Constraint." *European Journal of Operational Research*, May 1985, 255–60.

15. Reagan, Patricia B., "Inventory and Price Behaviour." *Review of Economic Studies*, January 1982, 137–42.

16. Schutte, David P., "Inventories and Sticky Prices: Note." *American Economic Review*, September 1983, 815–6.

Thompson, Gerald L., Suresh P. Sethi, and Teng Jinn-Tsair, "Strong Planning and Forecast Horizons for a Model with Simultaneous Price and Production Decisions." *European Journal of Operational Research*, 1984, 378–88.

18. Zabel, Edward, "Price Smoothing and Equilibrium in a Monopolistic Market." *International Economic Review*, June 1986, 349–63.