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A search for long memory in international stock market returns

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A major issue in financial economics is the behavior of stock returns over long as opposed to short horizons. This study provides empirical evidence from the perspective of long memory analysis. International evidence on long memory is explored using the Morgan Stanley Capital International stock index data for eighteen countries. Two tests that are robust to short-term dependence and conditional heteroskedasticity are employed: a modified rescaled range test and a fractional differencing test. The empirical results in general provide little support for long memory in international stock returns. The findings are not sensitive to inflation adjustments in stock returns, data sources, and statistical methods used. (JEL G12).

The behavior of stock returns over long as opposed to short horizons has been a hotly contested issue. Shiller (1984) and Summers (1986) show in simple models of fads that large and slowly decaying swings in stock prices away from fundamental values can occur, without significant autocorrelation in short-term returns. According to the Shiller–Summers models, market inefficiency can exist, but statistical tests on short-term returns will fail to detect it. Stambaugh (1986) observes, nonetheless, that the long swings away from fundamental values suggested by the Shiller–Summers models imply the presence of negative autocorrelation in long-horizon returns. This is because, to the extent that the swings away from fundamental values are not permanent, prices will be mean-reverting and returns will exhibit negative serial correlation over long horizons.

Fama and French (1988) examine autoregressions of multiperiod returns on diversified portfolio of NYSE stocks and report empirical findings seemingly consistent with the Shiller–Summers models. It is reported that while autocorrelations in returns are close to zero at short horizons, they become strongly

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negative at long horizons. The findings represent evidence against the random walk hypothesis. Following Lo and MacKinlay (1988), Poterba and Summers (1988) study the relationship between the variance of returns on diversified portfolio and the return horizon. Using variance ratio analysis, Poterba and Summers (1988) present evidence of positive autocorrelations in returns over short horizons but negative autocorrelations over long horizons. Fama and French (1988) note, however, that similar findings can be consistent with time-varying expected returns generated by efficient pricing. Other studies, on the other hand, question the statistical significance of these findings. Kim, Nelson and Shartz (1991), for example, observe that the previous findings are based on the assumption of normally distributed returns. Such an assumption is not valid in the presence of conditional heteroskedasticity. Richardson (1993) illustrates that analyses based merely on horizon-by-horizon statistics can yield misleading inferences for ignoring possible interdependence between different return horizons. Richardson and Stock (1989) further point out that the finite sample bias of multiperiod return regression tests can be significant. McQueen (1992) also emphasizes the need to correct for heteroskedasticity, correlations among returns at different horizons, and finite sample bias in applying the multiperiod return regression analysis.

An alternative approach to study the behavior of stock returns over long versus short horizons is explored by Lo (1991) based on long-memory analysis. The previous findings that stock returns exhibit positive autocorrelations over short horizons and negative autocorrelations over long horizons, if relevant, point to the presence of long cycles and hence a potentially predictable component in stock price dynamics. Lo (1991) observes that the reported anomalous behavior of stock returns can be a symptom of long-memory dynamics. A series having long memory is characterized by long-term dependence and non-periodic long cycles. Lo (1991) suggests the use of a modified rescaled range (R/S) procedure to detect long memory. The modified R/S test has a desirable property in that it is not sensitive to non-normality and conditional heteroskedasticity in the data. This R/S test is also robust to short-term dependence; consequently, the test can allow for a rich pattern of interactions between short- and long-term dynamics. Employing the modified R/S test, Lo (1991) examines US stock return data based on the value- and equally-weighted CRSP (Center for Research in Security Prices) indexes, and no significant evidence of long memory can be found in stock returns once short-term dependence and conditional heteroskedastic effects are accounted for. Certainly, the statistical failure to find long memory in stock returns does not absolutely deny its presence. If long memory in fact exists, however, new supportive evidence is needed to confirm its empirical relevance.

This study extends Lo's (1991) analysis by providing a more extensive and systematic study of stock market dynamics in several respects. First, international evidence concerning long memory in stock returns is explored. Since the time series evidence on long memory so far is limited, a search for some cross-country evidence seems useful. In this study we investigate whether Lo's (1991) finding is common to other national stock markets or unique to the US stock market. Stock market indexes in 17 other countries are considered: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore/Malaysia, Spain, Sweden, Switzerland and the United Kingdom. Secondly, in addition to modified R/S analysis, the fractional differencing test for long memory devised by Geweke and Porter-Hudak (1983) is employed to detect long memory in stock returns. The fractional differencing approach models long-memory dynamics parametrically, and it can potentially improve estimation efficiency upon the non-parametric R/S approach. The fractional differencing method has been mentioned, though not applied, by Lo (1991). Monte Carlo results reported by Cheung (1993) show that both the modified R/S test and Geweke and Porter-Hudak's (1983) fractional differencing test have reasonably good finite simple properties for at least moderate sample sizes. Thirdly, the dynamic behavior of both nominal and real stock return series is examined. While Lo's (1991) analysis considers nominal stock returns exclusively, the studies by Fama and French (1988), and Poterba and Summers (1988) examine real stock returns. In addition, a study by Diebold and Rudebusch (1989) reports that real national output displays long memory based on fractional differencing analysis. Since the extent to which stock price dynamics can reflect changes in economic fundamentals is still a debated issue, it is interesting to investigate if long memory is present in real stock returns as well. Fourthly, the behavior of stock dividend growth series is also studied. We investigate whether there is similar dynamic behavior between stock return series and dividend growth series. The latter series may provide indirect evidence on long-memory dynamics, if any, in stock markets.

I. Some related work on non-linear dependence

Analysis of non-linear dependence in stock prices has enjoyed much attention recently. An issue involved is whether the seemingly random movements in asset prices contain a detectable, albeit non-linear, structure. The growing interest in non-linear dynamics arises from the observation that the often wide and non-periodic cyclical fluctuations of stock prices cannot be adequately explained by linear models. The usual findings of leptokurtosis in stock returns may be further indirect evidence for non-linear dynamics. Scheinkman and LeBaron (1989) note that conditional heteroskedastic processes can display dependence resembling that of chaotic dynamics. Empirical evidence of chaotic dynamics or conditional heteroskedastic dependence in stock returns has been reported by Hsieh (1991), Mayfield and Mizrach (1992), and Scheinkman and LeBaron (1989).

The long-memory dynamics investigated in this study represent a special form of non-linear dynamics called 'fractal' (Mandelbrot, 1977). Fractal dynamics are an interesting form of non-linear dynamics, characterized by irregular cyclical fluctuations and long-term dependence. Fractal dynamics can in general be distinguished from conditional heteroskedasticity and chaotic dynamics. Conditional heteroskedasticity is non-linear dependence in conditional variance, *i.e.*, dependence in the second moment of the distribution of the time series variable, whereas fractal dynamics give non-linear dependence in the first moment of the distribution. Unlike conditional heteroskedasticity, moreover, fractal dynamics relate mainly to long-term rather than short-term dependence. On the other hand, findings of chaotic dynamics by themselves provide little information to researchers for modeling the underlying non-linearity because its specific structure still needs identification. In contrast, fractal dynamics, like conditional heteroskedasticity, can be modeled explicitly in a time series framework. Specifically, fractional processes can be used to model fractal dynamics.

Uncovering long-memory dynamics can have interesting implications. If long memory is indeed present in stock returns, statistical inferences concerning asset pricing models based on standard testing procedures may no longer be valid. Moreover, theoretical and empirical models that allow for long-memory price dynamics should be explored. Mandelbrot (1971) notes that, in the presence of long memory, the arrival of new market information cannot be fully arbitraged away and martingale models of asset prices cannot be obtained from arbitrage. Mandelbrot (1971) further shows that variability in the imperfectly arbitraged price may not be stationary and the return distribution is non-normal.

Empirical evidence on the presence of long memory in the US stock returns has first been presented by Greene and Fielitz (1977) based on the classical R/S analysis, first proposed by Hurst (1951) and later refined by Mandelbrot (1972), Mandelbrot and Wallis (1969), and Wallis and Matalas (1970), among others. Aydogan and Booth (1988) reexamine the evidence and suggest that the finding of long memory in the US stock returns may be spurious because of the existence of preasymptotic behavior in statistical estimates. Lo (1991) also observes that the classical R/S analysis is not robust to possible short-term dependence and heteroskedasticity in the data process. As a result, reliable statistical inferences are hard to make in the presence of either short-term dependence or heteroskedasticity or both. To minimize the problem, the study here employs two different tests for long memory—a modified R/S test and a fractional differencing test—which are robust to both short-term dependence and heteroskedasticity.

II. Empirical methodology

Both the modified R/S test and the fractional differencing test examine the null hypothesis of a short-memory process against long-memory alternatives. They are discussed briefly below.

II.A. Modified R / S analysis

The modified R/S test for long memory examines the null hypothesis of a short memory and possibly heteroskedastic process. Let $\bar{x}$ be the sample mean of a return data series $\{x_1, x_2, \ldots, x_T\}$. The modified R/S statistic, denoted by

Q_T, is given by the range of cumulative sums of deviations of the time series from its mean, rescaled by a consistent estimate of its standard deviation:

$$\langle 1\rangle \qquad Q_T = \left\{ \max_{1\le i\le T} \sum_{t=1}^{i} (x_t - \bar{x}) - \min_{1\le j\le T} \sum_{t=1}^{j} (x_t - \bar{x}) \right\} / s_T(q),$$

where $s_T^2(q)$ is a heteroskedasticity and autocorrelation consistent variance estimator (Andrews, 1991),

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$$s_T(q) = \left\{ \sum_{i=1}^{T} (x_i - \bar{x})^2/T + 2\sum_{j=1}^{q} \tau_j(q) \left(\sum_{i=j+1}^{T} (x_i - \bar{x})(x_{i-j} - \bar{x}) \right) / T \right\}^{1/2}$$

with the weighting function $\tau_j(q) = 1 - |j/z_T|$ and a truncation lag q determined by

$$\langle 3\rangle \qquad q = \mathrm{Int}[z_T], \qquad z_T = (3T/2)^{1/3}\{2\rho/(1-\rho^2)\}^{2/3},$$

where $\mathrm{Int}[z_T]$ denotes the integer part of z_T and ρ is the first-order autocorrelation of the data series.

The modified R/S statistic differs essentially from the classical one on the normalization of the range measure. The denominator in equation ⟨1⟩ normalizes the range measure not only by the sample variance ($q = 0$), as considered in the classical R/S analysis, but also by a weighted sum of sample autocovariances for $q > 0$. This modification provides the robustness of the modified R/S analysis to both short-term dependence and heteroskedasticity. Under the null hypothesis of no long memory, the limiting distribution of the Q_T statistic standardized by the square root of the sample size can be established. Critical values for the modified R/S test are tabulated by Lo (1991).

II.B. Fractional differencing analysis

Fractionally differenced processes explored by, *e.g.*, Granger and Joyeux (1980) and Hosking (1981) can be used to model parametrically long-memory dynamics. Under this approach, whether a series displays long memory depends on a fractional differencing parameter, which is amenable to estimation and hypothesis testing. A general class of long-memory process is described by

$$\langle 4\rangle \qquad B(L)(1-L)^d x_t = C(L)u_t,$$

where $B(L) = 1 - b_1L - \ldots - b_pL^p$ and $C(L) = 1 + c_1L + \ldots + c_qL^q$ are polynomials in the lag operator L, all roots of $B(L)$ and $C(L)$ are stable, and u_t is a white-noise disturbance term. The fractional parameter, given by d, assumes any real values. This fractional model includes the usual autoregressive moving-average (ARMA) model as a special case in which $d = 0$. The extension to have non-integer values of d raises the flexibility in modeling long-term dynamics by allowing for a rich class of spectral behavior at low frequencies. Granger and Joyeux (1980) and Hosking (1981) show that the spectral density function of x_t, denoted by $f_x(w)$, is proportional to w^{-2d} as w becomes small.

The fractional parameter thus crucially determines the low-frequency dynamics of the process.

A spectral method suggested by Geweke and Porter-Hudak (1983) can be used to estimate the fractional parameter d. The Geweke–Porter-Hudak (GPH) method provides a semi-non-parametric test for fractional processes that requires no explicit parameterization of the unknown ARMA dynamics. The statistical procedure involves estimating d using a spectral regression:

$$\langle 5\rangle \qquad \ln\big(I(w_j)\big) = \phi_0 - \phi_1 \ln\big(4\sin^2(w_j/2)\big) + \epsilon_t, \qquad j = 1, 2, \ldots, n,$$

where $I(w_j)$ is the periodogram at harmonic frequency $w_j = 2\pi j/T$, ϵ_t is a random error term, and $n = T^{\mu}$ for $0 < \mu < 1$ is the number of low-frequency ordinates used in the regression. The periodogram $I(w_j)$ is computed as the product of $2/T$ and the square of the exact finite Fourier transform of the series $\{x_1, x_2, \ldots, x_T\}$ at the respective harmonic ordinate. Geweke and Porter-Hudak (1983) show that the least squares estimate of ϕ_1 provides a consistent estimate of d and hypothesis testing concerning the value of d can be based on the usual t-statistic. The theoretical error variance for ϵ_t is known to be equal to $\pi^2/6$, which is typically imposed in estimation to raise efficiency.

III. Data and empirical results

This study considers eighteen national stock markets, those of Australia (AS), Austria (AT), Belgium (BE), Canada (CA), Denmark (DN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JA), Netherlands (NL), Norway (NO), Singapore/Malaysia (SX), Spain (SP), Sweden (SW), Switzerland (SZ), the United Kingdom (UK), and the United State (US). All data series consist of the monthly Morgan Stanley Capital International (MSCI) indexes, and they cover the sample period from January 1970 to August 1992. Unlike other available national stock market indexes, the MSCI indexes are fully comparable across countries since they are constructed on a consistent basis. The market indexes examined are value-weighted, computed with dividends reinvested, and expressed in terms of the US dollar and so adjusted for foreign exchange fluctuations. In constructing these indexes, the market values of investment companies and of foreign domiciled companies are excluded to avoid double-counting. Each price index series is transformed into a monthly return series by taking the ratio of the price change to the corresponding last-period price. Real returns are obtained from nominal returns by adjusting for the inflation rate in the US consumer price index. Cheung, Lai and Lai (1993), and Chow, Pan and Sakano (1992) study international evidence on long memory on stock returns using modified R/S analysis. Both studies consider nominal stock return data only, and use the stock market indexes reported in the IMF's International Financial Statistics. Since we have concerns about the quality of those data, we do not use them here.

Some preliminary data analysis of the stock return series is carried out, with attention paid to evidence concerning deviations from normality, autocorrelations, and conditional heteroskedasticity. Descriptive statistics for both nominal and real stock returns are presented in Table 1. Specifically, the skewness and

TABLE 1. Descriptive statistics for stock returns.

Country	Mean	s.d.	Min	Max	Skew	Kurt	$Q_{(5)}$	$Q_{(10)}$	$Q_{s(5)}$	$Q_{s(10)}$
(a) Nominal return series (in US$):										
AS	0.010	0.078	−0.445	0.255	−0.706*	4.876*	0.98	6.37	4.40	6.35
AT	0.012	0.064	−0.233	0.281	0.719*	3.526*	20.30*	29.20*	56.40*	99.20*
BE	0.014	0.059	−0.188	0.268	0.461*	2.886*	4.31	11.80	13.50*	16.30
CA	0.009	0.056	−0.220	0.180	−0.305*	2.039*	7.77	13.30	20.50*	25.10*
DN	0.013	0.056	−0.171	0.248	0.357*	1.365*	10.80	18.60*	0.91	6.85
FR	0.013	0.071	−0.232	0.268	−0.001	1.255*	6.26	10.60	7.47	9.43
GE	0.011	0.062	−0.176	0.202	−0.096	0.834*	7.17	19.00*	13.20*	21.10*
HK	0.023	0.119	−0.434	0.879	0.916*	10.526*	2.75	8.91	22.70*	23.10*
IT	0.007	0.076	−0.214	0.310	0.308*	1.041*	11.20*	22.30*	19.40*	23.00*
JA	0.015	0.067	−0.194	0.243	0.166	0.707*	4.26	10.60	8.50	15.20
NL	0.014	0.053	−0.178	0.257	−0.025	2.271*	4.50	14.50	4.07	7.23
NO	0.013	0.082	−0.279	0.254	−0.064	0.624*	12.30*	16.70	6.55	10.30
SX	0.016	0.090	−0.413	0.533	0.620*	6.647*	10.20	15.50	12.50*	14.60*
SP	0.009	0.066	−0.273	0.267	−0.032	2.091*	7.13	26.90*	15.40*	18.60*
SW	0.014	0.064	−0.214	0.202	−0.066	0.456	2.52	4.32	19.60*	22.70*
SZ	0.011	0.057	−0.176	0.246	0.064	1.489*	2.44	5.31	5.41	12.70
UK	0.013	0.077	−0.215	0.564	1.323*	9.549*	10.40	12.80	12.70*	13.70
US	0.010	0.046	−0.212	0.178	−0.197	2.218*	3.38	8.50	6.53	13.70
(b) Real return series (in real US$):										
AS	0.005	0.078	−0.448	0.245	−0.731*	4.790*	1.09	7.42	4.28	6.53
AT	0.007	0.064	−0.242	0.276	0.713*	3.507*	22.40*	32.60*	59.80*	94.50*
BE	0.009	0.060	−0.191	0.271	0.423*	2.883*	6.09	16.70	12.70*	15.70
CA	0.004	0.057	−0.237	0.178	−0.378*	2.136*	8.55	14.10	16.90*	22.10*
DN	0.008	0.057	−0.178	0.242	0.350*	1.280	13.90*	22.90*	0.64	8.02
FR	0.008	0.072	−0.240	0.264	−0.036	1.268*	7.49	12.00	8.24	11.90
GE	0.006	0.063	−0.185	0.201	−0.088	0.841*	7.59	19.30*	12.70*	21.70*
HK	0.018	0.120	−0.437	0.871	−0.877*	10.271*	5.71	9.69	24.50*	24.70*
IT	0.002	0.076	−0.217	0.313	0.300*	1.107*	16.60*	26.70*	19.10*	22.40*
JA	0.011	0.067	−0.199	0.237	0.184	0.705*	5.86	17.80	11.70	18.10
NL	0.009	0.054	−0.185	0.253	−0.056	2.223*	6.10	16.60	4.78	9.11
NO	0.008	0.082	−0.292	0.249	−0.110	0.661*	13.00*	17.00	5.60	11.10
SX	0.011	0.090	−0.416	0.529	0.600*	6.586*	13.30*	19.60*	13.20*	18.00
SP	0.004	0.066	−0.278	0.270	−0.003	2.142*	16.00*	32.50*	15.70*	17.10
SW	0.009	0.064	−0.223	0.200	−0.065	0.496	3.09	5.41	19.30*	23.00*
SZ	0.006	0.058	−0.179	0.242	0.029	1.475*	3.27	6.73	6.77	15.10
UK	0.008	0.077	−0.218	0.560	1.274*	9.307*	10.70	13.10	13.60*	15.10
US	0.005	0.047	−0.215	0.167	−0.230	1.939*	6.00	9.00	7.33	15.50

Notes: This table provides several descriptive statistics, including mean, standard deviation (s.d.), minimum, maximum, skewness, excess kurtosis, and Ljung–Box test statistics, for individual stock return series. The country symbols are described in the first paragraph of Section III. Statistical significance for skewness and kurtosis is evaluated using the Kendall–Stewart (1958) test. The columns beneath '$Q_{(m)}$' and '$Q_{s(m)}$' give, respectively, the Ljung–Box statistics for return series and squared return series for up to mth order serial correlation. Statistical significance is indicated by * at the 5% level.

excess kurtosis of the stock return distributions are computed. For a normal distribution, both skewness and excess kurtosis measures equal zero. Statistical significance for skewness and kurtosis is evaluated using the Kendall–Stuart (1958) test. In addition to the usual descriptive statistics, the Ljung–Box test statistics for checking serial correlation in the levels and the squares of the return series are given; they are denoted by $Q_{(m)}$ and $Q_{s(m)}$, respectively. These Ljung–Box statistics serve to provide evidence for possible dependence in the first and higher moments of the return distributions. Under the null hypothesis of no serial correlation, the $Q_{(m)}$ and $Q_{s(m)}$ statistics are distributed asymptotically as a chi-square distribution with m degrees of freedom.

As shown in Table 1, the nominal and real return data yield a similar pattern of results. In the case of skewness, the results appear mixed. For nine out of 18 series (AS, AT, BE, CA, DN, HK, IT, SX, and UK), the empirical return distributions are not significantly skewed. In the case of excess kurtosis, however, all but one (SW) series show significant excess kurtosis. The estimated excess kurtosis coefficients are all positive, implying that the empirical return distributions have a thicker tail than a normal distribution. The results generally support the presence of significant departures from normality in both nominal and real stock returns. The findings of non-normality may be symptomatic of non-linear dynamics. According to the $Q_{(m)}$ test statistics, moreover, there is significant evidence of short-term linear dependence for six (AT, DN, GE, IT, NO, and SP) nominal return series and seven (AT, DN, GE, IT, NO, SX, and SP) real return series. The $Q_{s(m)}$ test statistics, on the other hand, suggest the possible presence of non-linear dependence in higher moments for ten (AT, BE, CA, GE, HK, IT, SX, SP, SW, and UK) nominal and real return series.

More formal evidence on short-term dependence and conditional heteroskedasticity is obtained by modeling them directly in a time series framework. The ARCH model, introduced by Engle (1982), and its generalization called the GARCH model by Bollerslev (1986) have often been applied to model time-varying volatility in financial time series. In this study, a version of the GARCH model is fitted to the nominal and real stock return data. In general, an AR(r)-GARCH(p, q) model for a return process $\{x_t\}$ can be described by

$$x_t = \tau_0 + \sum_{i=1}^{r} \tau_i x_{t-i} + e_t, \qquad e_{t|t-1} \sim N(0, a_t), \tag{6}$$

$$a_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j e_{t-p}^2 + \sum_{k=1}^{q} \beta_k a_{t-k}, \tag{7}$$

where the model orders $r \geq 0$, $p \geq 0$, and $q \geq 0$; and the parameters $\alpha_0 \geq 0$, $\alpha_j \geq 0$, and $\beta_k \geq 0$ for non-negative variance. When $q = 0$, the model becomes an AR(r) – ARCH(p) process. In our empirical analysis, we first experimented with different combinations of model orders. We found that an AR(1)-GARCH(1, 1) model could fit the data best for most of the return series, though in some cases fitting an AR(1)-ARCH(1) model seemed adequate.

French, Schwert and Stambaugh (1987), for example, also find that the GARCH (1, 1) model can well capture heteroskedasticity in monthly stock return data. For the presentation here, estimation results for an AR(1)-GARCH(1, 1) model or an AR(1)-ARCH(1, 1) model, whichever fits the individual return series better, are reported. The model estimates are obtained using the Berndt–Hall–Hall–Hausman numerical algorithm.

Table 2 contains the estimation results for the ARCH and GARCH models. Residual diagnostic statistics are provided, but they are not based on regular residuals. The presence of conditional heteroskedasticity invalidates the standard asymptotic distribution theory of sample autocorrelations and hence of the Box–Ljung test statistics unless the series under examination has been adjusted for conditional heteroskedastic effects. One way to remove the heteroskedasticity is to construct a series of standardized residuals, obtained by dividing each residual by the corresponding estimated conditional standard deviation. The standardized residuals thus constructed can be used to check for any remaining dependence not captured by the estimated models. The columns beneath '$Q^*_{(m)}$' and '$Q^*_{s(m)}$' give the Ljung–Box test statistics for the standardized residuals ($\hat{e}_t/\hat{a}_t^{1/2}$) and the standardized squared residuals ($\hat{e}_t^2/\hat{a}_t$), respectively. In all but three (AT, GE, and SP) cases, the $Q^*_{(m)}$ statistics do not indicate any significant serial correlation in the standardized residuals. Furthermore, the $Q^*_{s(m)}$ statistics suggest that the standardized squared residuals from all the estimated models for nominal or real return series show no significant serial correlation, thus supporting the adequacy of the ARCH(1) or GARCH(1, 1) specification. According to the ARCH or GARCH parameter estimates, significant evidence of conditional heteroskedasticity shows up in 13 of the 18 nominal return series—the AS, AT, CA, FR, GE, HK, IT, JA, SX, SP, SW, UK, and US series. Significant, though not substantial, autoregressive dependence is also found in three cases (IT, NO, and SX). Adding these to the three other cases where the $Q^*_{(m)}$ statistics are statistically significant, there are six nominal returns series exhibiting short-term dependence. Qualitatively similar results are obtained for the real return series.

The results in Table 2 on the whole suggest the presence of either short-term dependence or conditional heteroskedasticity or both in most of the stock return data. In view of these results, it is desirable that tests for long memory should properly account for these relevant stochastic properties in the return data; otherwise, reliable statistical inferences cannot be drawn. In this regard, these results provide support for the use of the modified R/S test and the GPH fractional differencing test given their robustness to short-term dependence as well as conditional heteroskedasticity.

Cheung (1993) studies the sensitivity of several tests for long memory to short-term ARMA dependence, conditional heteroskedasticity, variance shifts, and mean shifts. In Monte Carlo experiments, both the modified R/S and the GPH tests are found to be robust to moderate ARMA dependence, variance shifts, and conditional heteroskedastic effects. These two tests are, however, sensitive to large ARMA components and mean shifts. Specifically, infrequent shifts in the process mean can cause both tests to yield spurious evidence of long memory. A large positive AR parameter (with a magnitude of 0.7 or

TABLE 2. Short-term dependence and conditional heteroskedasticity in stock returns.

Country	τ_0	τ_1	α_0	α_1	β_1	$Q^*_{(5)}$	$Q^*_{(10)}$	$Q^*_{s(5)}$	$Q^*_{s(10)}$
(a) Nominal return series (in US$):									
AS	0.007 (0.004)	−0.020 (0.075)	0.001 (0.000)*	0.129 (0.074)*	0.670 (0.188)*	1.66	6.29	0.56	1.81
AT	0.007 (0.003)*	0.087 (0.065)	0.001 (0.000)*	0.147 (0.028)*	0.832 (0.030)*	11.80*	18.20	6.21	6.48
BE	0.013 (0.004)*	0.089 (0.068)	0.003 (0.001)*	0.067 (0.050)		1.99	10.40	10.70	12.90
CA	0.010 (0.003)*	−0.023 (0.071)	0.001 (0.000)*	0.140 (0.066)*	0.598 (0.188)*	5.90	9.90	2.24	13.70
DN	0.013 (0.003)*	0.021 (0.021)	0.003 (0.001)*	0.019 (0.073)		9.66	17.10	0.06	6.34
FR	0.012 (0.004)*	0.098 (0.067)	0.004 (0.000)*	0.127 (0.071)*		4.12	9.76	5.86	7.05
GE	0.009 (0.004)*	−0.003 (0.067)	0.003 (0.001)*	0.233 (0.094)*		8.32	18.60*	4.82	9.02
HK	0.019 (0.005)*	0.030 (0.077)	0.001 (0.000)*	0.248 (0.055)*	0.714 (0.080)*	2.06	7.88	2.24	4.54
IT	0.004 (0.005)	0.115 (0.048)*	0.005 (0.001)*	0.196 (0.101)*		4.55	13.70	7.26	9.75
JA	0.016 (0.004)*	0.084 (0.068)	0.001 (0.001)	0.156 (0.069)*	0.605 (0.195)*	3.54	8.00	1.07	5.31
NL	0.013 (0.003)*	0.030 (0.067)	0.003 (0.000)*	0.030 (0.054)		4.44	14.00	3.41	7.74
NO	0.011 (0.005)*	0.155 (0.064)*	0.006 (0.001)*	0.067 (0.072)		9.38	14.40	0.24	2.70
SX	0.007 (0.004)	0.217 (0.087)*	0.002 (0.001)*	0.284 (0.055)*	0.748 (0.033)*	7.51	11.80	0.52	2.08
SP	0.006 (0.004)	0.093 (0.071)	0.001 (0.001)	0.120 (0.061)*	0.623 (0.224)*	1.85	19.50*	0.25	2.80
SW	0.012 (0.004)*	0.040 (0.071)	0.001 (0.001)	0.130 (0.062)*	0.681 (0.188)*	1.31	4.17	2.74	8.09
SZ	0.011 (0.003)*	0.034 (0.066)	0.003 (0.000)*	0.080 (0.067)		1.97	4.57	5.04	10.20
UK	0.015 (0.004)*	0.003 (0.079)	0.001 (0.000)*	0.101 (0.050)*	0.815 (0.105)*	3.44	6.64	1.32	4.29
US	0.011 (0.003)*	−0.005 (0.072)	0.002 (0.001)*	0.091 (0.054)*		3.43	8.96	1.18	9.81

TABLE 2. (Cont.)

Country	τ_0	τ_1	α_0	α_1	β_1	$Q^*_{(5)}$	$Q^*_{(10}$	$Q^*_{s(5)}$	$Q^*_{s(10)}$
(b) Real return series (in real US$):									
AS	0.003	−0.012	0.001	0.118	0.689	1.62	6.45	0.66	1.77
	(0.004)	(0.076)	(0.001)	(0.069)*	(0.186)*				
AT	0.002	0.100	0.001	0.139	0.840	12.50*	19.4*	6.32	6.65
	(0.003)	(0.064)	(0.000)*	(0.027)*	(0.029)*				
BE	0.008	0.109	0.003	0.064		2.36	11.70	9.30	11.40
	(0.004)*	(0.067)	(0.001)*	(0.050)					
CA	0.005	−0.021	0.001	0.139	0.602	6.27	10.10	2.61	13.80
	(0.003)	(0.072)	(0.000)*	(0.064)*	(0.187)*				
DN	0.008	0.037	0.003	0.015		10.50	17.50	0.09	6.86
	(0.004)*	(0.063)	(0.000)*	(0.069)					
FR	0.008	0.110	0.004	0.140		4.65	10.30	5.21	6.45
	(0.004)*	(0.068)	(0.000)*	(0.074*					
GE	0.005	0.015	0.003	0.219		8.47	18.90*	4.57	8.20
	(0.004)	(0.067)	(0.001)*	(0.092)*					
HK	0.014	0.038	0.001	0.243	0.718	1.86	7.99	2.21	4.49
	(0.006)*	(0.077)	(0.000)*	(0.054)*	(0.081)*				
IT	−0.001	0.118	0.005	0.198		4.60	13.20	7.71	9.87
	(0.004)	(0.048)*	(0.001)*	(0.102)*					
JA	0.012	0.101	0.001	0.156	0.591	4.05	9.47	1.02	4.70
	(0.004)*	(0.068)	(0.001)	(0.068)*	(0.203)*				
NL	0.009	0.046	0.003	0.027		4.18	13.60	3.80	8.72
	(0.003)*	(0.067)	(0.000)*	(0.053)					
NO	0.007	0.153	0.006	0.064		9.66	14.70	0.35	2.74
	(0.005)	(0.064)*	(0.001)*	(0.072)					
SX	0.004	0.213	0.002	0.285	0.748	7.25	11.10	0.51	2.13
	(0.004)	(0.088)*	(0.001)*	(0.057)*	(0.034)*				
SP	0.002	0.097	0.001	0.121	0.608	1.70	20.80	0.30	2.65
	(0.004)	(0.072)	(0.001)	(0.061)*	(0.229)*				
SW	0.007	0.046	0.001	0.124	0.677	1.44	4.42	3.16	9.12
	(0.004)	(0.071)	(0.001)	(0.061)*	(0.202)*				
SZ	0.006	0.055	0.003	0.076		1.89	5.27	5.81	11.50
	(0.003)*	(0.066)	(0.000)*	(0.064)					

TABLE 2. (Cont.)

Country	τ_0	τ_1	α_0	α_1	β_1	$Q^*_{(5)}$	$Q^*_{(10)}$	$Q^*_{s(5)}$	$Q^*_{s(10)}$
UK	0.011	0.004	0.005	0.104	0.818	3.28	6.70	1.71	5.37
	(0.004)*	(0.080)	(0.003)	(0.045)*	(0.099)*				
US	0.006	0.015	0.002	0.099		3.77	8.25	1.20	10.20
	(0.003)*	(0.071)	(0.001)*	(0.057)*					

Notes: The AR(1)-GARCH (1, 1) model is fitted to individual stock return series (the country symbols are described in the first paragraph of Section III). The estimated model is given by

$$x_t = \tau_0 + \tau_1 x_{t-1} + e_t, \qquad e_{t|t-1} \sim N(0, a_t)$$

$$a_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 a_{t-1}.$$

In some cases, an AR(1)-ARCH(1) model (with $\beta_1 = 0$) is found to fit the data better, so the results for the ARCH model are reported instead. The asymptotic standard errors are in parentheses under the corresponding coefficient estimates. The columns beneath '$Q_{(m)}$' and '$Q^*_{s(m)}$' provide, respectively, the Ljung–Box statistic for the standardized residuals ($\hat{e}_t/\hat{a}_t^{1/2}$) and their squares ($\hat{e}_t^2/\hat{a}_t$) for up to mth order serial correlation; the asymptotic distribution of these statistics is $\chi^2(m)$ under the null hypothesis of no serial correlation.

Statistical significance is indicated by an asterisk (*) for the 5% level.

higher) can also bias the GPH test and, to a lesser extent, the modified R/S test toward finding long memory too often. Agiakloglou, Newbold and Wohar (1993) examine the performance of the GPH test and note also the significant bias in the test when there is a large positive AR parameter. A similar point has been observed by Sowell (1992) as well.

The foregoing studies commonly note the possible test bias against the no-long-memory null hypothesis induced by strong short-term dependence. This does not pose any serious problem to our stock return analysis here, however. The short-term dependence found in stock return series is apparently small (the largest AR coefficient estimate obtained from our series is about 0.2 only). In addition, the general finding reported later in this paper fails to suggest the presence of significant long memory in stock returns; whereas, the bias, if relevant, should presumably lead to frequent rejections of the hypothesis of no long memory.

The modified R/S test is performed on both the nominal and real return series, and the results are reported in Table 3. The modified R/S test statistics are provided together with the optimal lag selected and used in constructing the corresponding statistics. The results for nominal stock returns uniformly show that in no case can the null hypothesis of no long memory be rejected at the 5 percent significance level. The results for real stock returns differ very little from those for nominal stock returns. In only one case (SP) can the null hypothesis of no long memory be rejected at the 5 percent significance level.

TABLE 3. Results of modified R/S analysis for stock return series.

(a) Nominal return series:			*(b) Real return series:*		
Country	R/S statistic	q-Lag selected	Country	R/S statistic	q-Lag selected
AS	1.158	0	AS	1.116	0
AT	1.696	3	AT	1.764	3
BE	1.456	2	BE	1.654	2
CA	0.871	0	CA	0.996	0
DN	1.280	0	DN	1.468	1
FR	1.013	2	FR	1.131	2
GE	1.129	0	GE	1.283	0
HK	1.351	1	HK	1.460	2
IT	1.635	3	IT	1.571	3
JA	1.285	1	JA	1.372	2
NL	1.217	1	NL	1.337	1
NO	1.046	3	NO	1.188	3
SX	1.049	3	SX	1.142	3
SP	1.745	2	SP	1.893*	2
SW	1.301	2	SW	1.401	2
SZ	0.948	1	SZ	1.059	2
UK	1.228	2	UK	1.174	2
US	1.118	1	US	1.273	1

Notes: The modified R/S test for long memory suggested by Lo (1991) is performed on the stock return series for each of the following countries: Australia (AS), Austria (AT), Belgium (BE), Canada (CA), Denmark (DN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JA), Netherlands (NL), Norway (NO), Singapore/Malaysia (SX), Spain (SP), Sweden (SW), Switzerland (SZ), the United Kingdom (UK), and the United States (US). The lag parameter q used for the modified R/S test is determined by Andrew's (1991) data-dependent rule (see equation ⟨3⟩). At the 5% significance level, the null hypothesis of a short-memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862].

Statistical significance is indicated by an asterisk (*) for the 5% level.

These generally negative findings from the modified R/S test do not support the presence of long memory in either nominal or real stock returns.

The GPH fractional differencing test is next performed on the stock return data. With a potential gain in estimation efficiency, the fractional differencing test may provide a more powerful test for long memory than the modified R/S test. The fractional differencing test serves to uncover fractal structure in a time series based on spectral analysis of its low-frequency dynamics. In applying the GPH spectral procedure, the number of low-frequency ordinates, n, used in the spectral regression is a choice variable. The choice necessarily involves judgement. While a too large value of n will cause contamination of the d estimate due to medium- or high-frequency components, a too small value of n will lead to imprecise estimates due to limited degrees of freedom in estimation. To balance these two consideration factors, we experiment with a range of μ values used for the sample size function, $n = T^{\mu}$. The results reported below are for $\mu = 0.50$ and 0.55. This selected set of μ values yields good test performance in our experiment.

Table 4 contains the estimates for the fractional parameter d from the GPH spectral regression. The d estimates are reported together with their t-statistics. Significant evidence of long memory can be found in only four (AT, IT, JA, and SP) cases of nominal returns and five (AT, BE, IT, JA, and SP) cases of real returns. In comparison with the earlier results from the modified R/S analysis, the fractional differencing analysis provides slightly more favorable

TABLE 4. Results of fractional differencing analysis for stock return series.

(a) Nominal return series:					*(b) Real return series:*				
Country	$n = T^{.50}$		$n = T^{.55}$		Country	$n = T^{.50}$		$n = T^{.55}$	
	d	(t-stat)	d	(t-stat)		d	(t-stat)	d	(t-stat)
AS	0.214	(1.016)	0.027	(0.155)	AS	0.226	(1.074)	0.020	(0.113)
AT	0.330	(1.570)	0.386	(2.193)*	AT	0.378	(1.796)	0.419	(2.318)*
BE	0.383	(1.821)	0.303	(1.723)	BE	0.457	(2.176)*	0.353	(2.008)*
CA	−0.043	(−0.206)	−0.100	(−0.566)	CA	−0.078	(−0.373)	−0.128	(−0.730)
DN	0.069	(0.329)	0.123	(0.701)	DN	0.166	(0.788)	0.201	(1.142)
FR	0.032	(0.151)	0.070	(0.400)	FR	0.154	(0.502)	−0.001	(−0.006)
GE	0.086	(0.409)	0.103	(0.585)	GE	0.184	(0.875)	0.173	(0.984)
HK	0.019	(0.091)	−0.018	−0.102	HK	0.040	(0.190)	−0.004	(−0.023)
IT	0.560	(2.663)*	0.400	(2.272)*	IT	0.572	(2.721)*	0.410	(2.330)*
JA	0.468	(2.224)*	0.410	(2.329)*	JA	0.533	(2.537)*	0.451	(2.563)*
NL	0.311	(1.480)	0.067	(0.383)	NL	0.438	(2.083)	0.163	(0.926)
NO	−0.051	(−0.241)	0.037	(−0.212)	NO	0.046	(0.217)	0.028	(0.156)
SX	0.031	(0.149)	0.095	(0.537)	SX	−0.014	(−0.067)	0.052	(0.297)
SP	0.522	(2.481)*	0.418	(2.378)*	SP	0.543	(2.583)*	0.443	(2.529)*
SW	0.159	(0.756)	0.173	(0.980)	SW	0.203	(0.966)	0.205	(1.167)
SZ	0.168	(0.800)	0.149	(0.844)	SZ	0.330	(1.570)	0.263	(1.494)
UK	−0.109	(−0.516)	−0.044	(−0.251)	UK	−0.065	(−0.311)	−0.002	(−0.011)
US	−0.001	(−0.007)	−0.148	(−0.841)	US	−0.014	(−0.066)	−0.131	(−0.746)

Notes: The spectral fractional test for long memory suggested by Geweke–Porter-Hudak (1983) is performed on the stock return series for each of the following countries: Australia (AS), Austria (AT), Belgium (BE), Canada (CA), Denmark (DN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JA), Netherlands (NL), Norway (NO), Singapore/Malaysia (SX), Spain (SP), Sweden (SW), Switzerland (SZ), the United Kingdom (UK), and the United States (US). The number of low-frequency ordinates (n) used in the GPH spectral regression is given by $n = T^{\mu}$, where $T = 272$ is the effective number of observations. The figures in parentheses are the t-statistics for the corresponding fractional parameter d estimates, computed based on the known theoretical error variance ($\pi^2/6$).

Statistical significance in finding long memory is indicated by an asterisk (*) for the 5% level.

evidence for long memory in stock returns. The evidence for long memory is, nevertheless, still limited and far from pervasive.

IV. Evidence from stock dividend series

The behavior of stock dividend growth series is next investigated and compared with the stock return series in the hope of finding indirect supplementary evidence on long memory. According to the usual present value model, the value of a stock is determined by the discounted value of the dividend stream paid out by the firm, and stock price changes will be related to changes in dividends.

The dividend series examined are based on the MSCI data. They are obtained from multiplying the difference between dividend-inclusive and dividend-exclusive returns by the previous month's price index. These dividend series represent the gross dividends an investor would receive if the investor were investing in the portfolios defined by the Capital International stock price indexes. All the dividend series have been adjusted for rights issues, stock dividends, and stock splits.

The results of the modified R/S test for long memory in both nominal and real dividend growth series are summarized in Table 5. For the nominal dividend growth series, the null hypothesis of no long memory cannot be rejected at the 5 percent significance level in any of the cases considered. Similar results are obtained for the real dividend series. In only one case (DN) can the null hypothesis of no long memory be rejected at the 5 percent level. Table 6 reports the corresponding results of fractional differencing analysis. Again for the nominal dividend growth series, no significant evidence of long memory can be found in any of the cases under consideration. For the real dividend growth series, the hypothesis of no long memory cannot be rejected statistically in all but two cases (CA and US). The overall evidence is clearly not favorable to the hypothesis of long memory.

V. Conclusions

Financial economists continue to seek a better understanding of the nature of stock price dynamics. A major concern is the behavior of stock returns over long as opposed to short horizons. Several earlier studies report empirical evidence suggesting that stock returns exhibit positive serial correlation over short horizons but negative serial correlation over longer horizons. The reported evidence is based either on autoregression analysis of multiperiod returns (Fama and French, 1988) or variance ratio analysis (Lo and MacKinlay, 1988; Poterba and Summers, 1988).

This study attempts to contribute alternative evidence from the perspective of long memory analysis, motivated by Lo's (1991) work. International evidence on long memory is examined based on both nominal and real stock return data from eighteen countries. Two relatively new tests for long memory are employed, Lo's (1991) modified R/S test and Geweke and Porter-Hudak's (1983) fractional differencing test. These two tests have the attractive properties that

TABLE 5. Results of modified R/S analysis for dividend growth series.

(a) Nominal dividend series:			*(b) Real dividend series:*		
Country	R/S statistic	q-Lag selected	Country	R/S statistic	q-Lag selected
AS	1.400	0	AS	1.558	0
AT	1.032	2	AT	1.032	2
BE	1.555	1	BE	1.704	1
CA	1.138	2	CA	1.233	3
DN	1.670	5	DN	1.876*	1
FR	1.042	9	FR	1.041	9
GE	1.039	2	GE	1.039	2
HK	1.232	3	HK	1.232	3
IT	1.547	1	IT	1.677	1
JA	1.051	1	JA	1.232	0
NL	1.203	4	NL	1.201	4
NO	0.868	0	NO	0.874	0
SX	1.561	3	SX	1.858	3
SP	1.265	6	SP	1.253	6
SW	1.297	2	SW	1.462	2
SZ	0.838	2	SZ	0.947	2
UK	1.090	7	UK	1.285	7
US	1.662	3	US	1.372	3

Notes: The modified R/S test for long memory suggested by Lo (1991) is performed on the stock dividend series for each of the following countries: Australia (AS), Austria (AT), Belgium (BE), Canada (CA), Denmark (DN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JA), Netherlands (NL), Norway (NO), Singapore/Malaysia (SX), Spain (SP), Sweden (SW), Switzerland (SZ), the United Kingdom (UK), and the United States (US). The lag parameter q used for the modified R/S test is determined by Andrews's (1991) data-dependent rule (see equation ⟨3⟩). At the 5% significance level, the null hypothesis of a short-memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862].

Statistical significance is indicated by an asterisk (*) for the 5% level.

they are robust to non-normality and to the conditional heteroskedasticity that characterizes stock returns, and that they can allow for a rich pattern of interactions between short-term and long-term price dynamics. The results from the modified R/S test are mostly negative: little significant evidence for long memory can be found in either nominal or real stock return data. The fractional differencing test yields slightly more positive evidence than the modified R/S test, but the evidence in favor of the presence of long memory is far from pervasive. In only five out of the 18 countries considered can significant evidence of long memory be found. Further analysis of the behavior of dividend growth series also fails to yield indirect evidence for long memory. The results on the whole are not supportive of the presence of long memory in stock returns.

TABLE 6. Results of fractional differencing analysis for dividend growth series.

(a) Nominal dividend series:					*(b) Real dividend series:*				
Country	$n = T^{.50}$		$n = T^{.55}$		Country	$n = T^{.50}$		$n = T^{.55}$	
	d	(t-stat)	d	(t-stat)		d	(t-stat)	d	(t-stat)
AS	0.229	(1.091)	0.230	(1.307)	AS	0.363	(1.727)	0.305	(1.731)
AT	0.037	(0.174)	0.017	(0.095)	AT	0.037	(0.177)	0.016	(0.091)
BE	0.249	(1.182)	0.262	(1.490)	BE	0.297	(1.414)	0.293	(1.654)
CA	−0.100	(−0.476)	−0.102	(−0.581)	CA	0.207	(0.985)	0.352	(2.001)*
DN	0.299	(1.420)	0.166	(0.946)	DN	0.353	(1.679)	0.202	(1.150)
FR	−0.023	(−0.109)	0.003	(0.018)	FR	−0.010	(−0.049)	0.013	(0.073)
GE	−0.033	(−0.157)	−0.032	(−0.181)	GE	−0.037	(−0.177)	−0.035	(−0.198)
HK	−0.014	(−0.064)	−0.073	(−0.417)	HK	−0.007	(−0.102)	−0.080	(−0.454)
IT	−0.186	(−0.886)	−0.033	(−0.187)	IT	−0.097	(−0.462)	0.041	(0.232)
JA	0.035	(0.168)	−0.029	(−0.164)	JA	0.190	(0.902)	0.078	(0.443)
NL	−0.149	(−0.709)	−0.071	(−0.404)	NL	−0.148	(−0.702)	−0.066	(−0.376)
NO	−0.374	(−1.779)	−0.327	(−1.857)	NO	−0.229	(−1.090)	−0.223	(−1.266)
SX	0.347	(1.641)	0.208	(1.182)	SX	0.298	(1.415)	0.153	(0.870)
SP	−0.399	(−1.900)	−0.278	(−1.581)	SP	−0.264	(−1.255)	−0.168	(−0.954)
SW	0.098	(0.464)	−0.258	(−1.229)	SW	0.140	(0.666)	0.204	(1.160)
SZ	−0.258	(−1.229)	−0.092	(−0.523)	SZ	−0.100	(−0.476)	0.023	(0.133)
UK	0.091	(0.431)	0.048	(0.271)	UK	0.125	(0.592)	0.082	(0.463)
US	0.268	(1.277)	0.220	(1.250)	US	0.341	(1.623)	0.349	(1.986)*

Notes: The spectral fractional differencing test for long memory suggested by Geweke–Porter Hudak (1983) is performed on the stock divident series for each of the following countries: Australia (AS), Austria (AT), Belgium (BE), Canada (CA), Denmark (DN), France (FR), Germany (GE), Hong Kong (HK), Italy (IT), Japan (JA), Netherlands (NL), Norway (NO), Singapore/Malaysia (SX), Spain (SP), Sweden (SW), Switzerland (SZ), the United Kingdom (UK), and the United States (US). The number of low-frequency ordinates, n, used in the GPH spectral regression is given by $n = T^{\mu}$, where $T = 271$ is the effective number of observations. The figures in parentheses are the t-statistics for the corresponding fractional parameter d estimates, computed based on the known theoretical error variance ($\pi^2/6$).

Statistical significance in finding long memory is indicated by an asterisk (*) for the 5% level.

The findings here reinforce those reported by Lo (1991), and Cheung, Lai and Lai (1993). Lo (1991), employing the modified R/S test, does not find supportive evidence for long memory in US nominal stock returns based on CRSP data. Cheung *et al.* (1993) extends Lo's analysis and conclusion to four major foreign markets using the IMF's *International Financial Statistics* data. The present study further shows that the inflation adjustments involved in using real rather than nominal stock returns do not change the conclusion much. Neither do the data sources (IMF or MSCI) nor the statistical methods (modified R/S or fractional differencing).

A caveat is in order. As in previous empirical studies, analyzing long-term dependence in any time series incurs a typical problem. A statistical test may have low power to detect the dependence given the limited time span of data available. The lack of availability of long historical stock return data for many countries restricts our empirical study. Lo (1991) examines annual US stock return data based on the dividend adjusted S&P Composite Index from 1872 to 1986, and cannot find any significant evidence of long memory. Using the S&P daily closing price index from January 1928 through August 1991, on the other hand, Ding, Granger and Engle (1993) investigate the temporal dependence in the conditional variance of stock returns and find some evidence of long memory. In the presence of long-memory conditional heteroskedasticity, the time series, though still strongly stationary and ergodic, is not covariance stationary because of an infinite unconditional second moment. Since little is known about its effects on the modified R/S and GPH tests, future research on the robustness of these tests to long-memory conditional heteroskedasticity is warranted.

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