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The puzzling unit root in the real interest rate and its inconsistency with intertemporal consumption behavior

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Abstract

Previous studies of real interest rates generally have great difficulty rejecting unit-root dynamics, especially for industrial countries. The apparent unit-root behavior is puzzling because it contradicts both standard intertemporal asset pricing models and the Fisher effect. In examining international data for both industrial and developing countries, this study uncovers new evidence supporting the Fisher effect. It shows that structural change in real interest rate dynamics can be responsible for the observed unit-root behavior. When a mean shift is permitted under the alternative hypothesis, strong evidence against unit-root dynamics is unveiled for both industrial and developing countries. The cross-country findings provide wide support for the Fisher effect and resolve the puzzling inconsistency with intertemporal consumption behavior.

JEL classification: F43; F32

Keywords: Real interest rate; Asset pricing; Fisher effect; Structural break; Mean shift

1. Introduction

The real interest rate plays an important role in influencing saving and investment decisions. These influences constitute the central building block of consumption-based

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intertemporal models of asset prices (Lucas, 1978; Breeden, 1979; Hansen and Singleton, 1983), which are popular in the theoretical literature on capital asset pricing. These models imply that real interest rates and consumption growth rates should share similar stationarity properties. The observed behavior of real interest rates, however, appears inconsistent with intertemporal consumption behavior. As first noted by Rose (1988), real interest rates are found to be nonstationary with a unit root, while consumption growth rates contain no unit root.

The unit-root finding for real interest rates is puzzling because it contradicts not only consumption-based asset pricing models but also the Fisher effect – one of the oldest equilibrium asset pricing conditions. The Fisher (1930) hypothesis, in accordance with the classical theory of long-run neutrality, suggests that changes in inflation expectations are fully reflected in nominal interest rate adjustment. For the Fisher effect to hold, the real interest rate – the difference between the nominal interest rate and the expected inflation rate – should display long-run reversion. If a unit root is present, the real interest rate will have little or no tendency to revert to its equilibrium value. The unit-root finding thus seriously calls into question the empirical relevance of the long-run Fisher effect.

Since Rose (1988), many studies have reexamined the Fisher effect and continued to find it hard to establish its validity empirically (e.g., Mishkin, 1995; Evans and Lewis, 1995; Crowder and Hoffman, 1996; Daniels et al., 1996; Lai, 1997a; Lee et al., 1998; Koustas and Serletis, 1999; Lanne, 2001; Atkins and Coe, 2002). These studies largely report less than supportive evidence, especially in cross-country data for industrial countries (Koustas and Serletis, 1999). Although the nominal interest rate and the inflation rate do tend to move in the same direction over time, the former does not fully adjust to the latter; consequently, they fail to maintain a one-for-one adjustment relationship over the long run. In other words, the real interest rate, which measures the deviation from the equilibrium Fisher relationship, exhibits little long-run reversion.

This study presents new evidence in support of the long-run Fisher effect. The study identifies the presence of nonlinear dynamics in real interest rates as the key explanation for the widespread empirical failure to reject the unit-root hypothesis. Specifically, the puzzling unit-root behavior of real interest rates may stem from structural breaks. Although structural breaks are rather infrequent events, they can create the appearance that permanent shocks are predominant even when most shocks are actually transitory. As a result, the presence of structural breaks can confound unit-root tests and cause them to incorrectly fail to detect long-run reversion. The consideration of the structural-break possibility is not entirely new. Evans and Lewis (1995), Garcia and Perron (1996), Malliaropulos (2000), and Lai (2004) have illustrated the applicability of structural-break models to real interest rates for the U.S. The present study examines the general relevance of the structural-break explanation to countries beyond the U.S. By allowing for a structural break, broad international evidence against the unit-root hypothesis can be found.

This study broadens the analysis by including both industrial and developing countries. Previous studies on the Fisher effect have focused primarily on the U.S. and other industrial countries. Empirical evidence for developing countries, by contrast, is very much limited. A question can be raised about whether the existing results for industrial countries may overstate or understate the empirical relevance of the Fisher effect. Obviously, economists are interested in the applicability of the Fisher relationship to not only industrial but also developing countries. Moreover, if the absence of the Fisher effect is any indication of market inefficiency, we expect to have even greater difficulty finding the Fisher effect for developing countries than for industrial countries because the latter should have more efficient capital markets. In search of international evidence, this study finds that it is actually more difficult, rather than less difficult, to uncover the Fisher effect for industrial than for developing countries. It follows that the widespread failure to establish the Fisher effect cannot be explained by market inefficiency. Something else should be responsible.

In addition to the structural-break explanation, this study evaluates three other possible explanations for the common difficulty in finding empirical support for the Fisher effect. First, researchers often complain that standard unit-root tests are too inefficient and lacking in statistical power to reject the unit-root hypothesis. Second, having the unit root as the maintained hypothesis, standard unit-root tests by design make the unit-root hypothesis hard to reject unless there is very strong evidence against it. This can lead to under-rejections of the unit-root hypothesis. Third, long-memory dynamics may exist, foiling statistical tests in their ability to detect stationarity. This study shows that none of these three alternative explanations can fully account for the widespread failure to reject the unit-root hypothesis. Only the structural-break explanation is found to be adequate for unveiling broad evidence in favor of no unit root.

2. Consumption-based asset pricing models and the Fisher effect

Standard consumption-based asset pricing models suggest that the equilibrium price of an asset is determined by the expected present value of its future returns, adjusted by the intertemporal marginal rate of substitution in consumption. Consider a representative agent who chooses consumption and investment plans so as to maximize the expected value of a time-additive utility function:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(C_t)\right], \quad 0<\beta<1 \tag{1}$$

where β is a time preference parameter, C_t is consumption at time t, and E_t denotes expectations conditioned on information available at time t. The utility maximization is under the budget constraint:

$$(q_{t-1}+d_{t-1})W_{t-1}=q_{t-1}W_t+C_{t-1} \tag{2}$$

where W_t is the holdings of assets (measured in units of consumption goods) at time t, q_t is the vector of prices corresponding to the different assets in W_t, and d_t denotes the vector of values of the distributions.

The first-order Euler condition (Lucas, 1978) is given by

$$E_{t-1}[\beta U'(C_t)\,(1+r_{i,t})]=U'(C_{t-1}) \quad \forall i \tag{3}$$

where $U'(C_t)$ is the marginal utility of consumption and $r_{i,t}=(q_{i,t}+d_{i,t})/q_{i,t-1}-1$ is the real rate of return on asset i in period t. To derive testable implications, consider the usual utility function with constant relative risk aversion:

$$U(C_t)=C_t^{1-\theta}/(1-\theta), \quad \theta>0 \tag{4}$$

The first-order optimality condition in Eq. (3) can then be simplified to

$$E_{t-1}[\beta(C_t/C_{t-1})^{-\theta}(1+r_{i,t})] = 1 \tag{5}$$

Following Hansen and Singleton (1983), an empirical representation of this equilibrium condition assumes a log-linear form:

$$\ln(\beta) - \theta E_{t-1}[\Delta\ln(C_t)] + E_{t-1}[\ln(1+r_{i,t})] = 0 \tag{6}$$

This suggests that the time-series properties of $\Delta\ln(C_t)$ and $\ln(1+r_{i,t})$, which represent respectively the real consumption growth rate and the log of real asset returns, should be similar. Specifically, whether real asset returns contain a unit root brings into question the validity of consumption-based asset pricing models. According to the empirical evidence reported by Rose (1988) for U.S. data, the real consumption growth rate contains no unit root but short-term real bill returns have a unit root – a result inconsistent with the equilibrium asset pricing condition described by Eq. (6).

The unit-root issue in the real interest rate also bears on the validity of the Fisher effect. According to the Fisher equation, the one-period nominal interest rate at time t (denoted by i_t) has two components:

$$i_t = r_t + \pi_t^e \tag{7}$$

where r_t is the *ex ante* real interest rate and π_t^e is the expected inflation rate. If changes in expected inflation have no permanent impact on r_t, in accordance with the classical long-run neutrality theory, they should be reflected fully in subsequent movements of i_t over time. This implies a one-to-one relationship between the nominal interest rate and expected inflation in the long run. For the long-run Fisher relationship to hold, the real interest rate – the difference between the nominal interest rate and expected inflation – should be a stationary process. If r_t is stationary, $\ln(1+r_t)$ is also stationary. If r_t has a unit root, on the other hand, $\ln(1+r_t)$ will share the same nonstationary component. Hence, the validity of consumption-based asset pricing models is closely intertwined with the validity of the Fisher effect.

Testing for a unit root in the *ex ante* real interest rate requires some measure of expected inflation, which is unobservable. The usual approach uses realized inflation as a proxy for expected inflation to compute the *ex post* rate. The difference between the *ex post* rate and the *ex ante* rate reflects the inflation forecast error. To the extent that the inflation forecast error follows a stationary process, the *ex post* rate should exhibit the same unit-root property as the *ex ante* rate. This leads to the caveat that a test for a unit root in the *ex post* real interest rate represents a test of the joint hypothesis of the Fisher effect and the stationarity of inflation forecast errors. Test results should be interpreted with the caveat.

3. The data

This study examines the behavior of real interest rates and real consumption growth rates for 16 countries, including eight major industrial countries (Australia, Canada, France, Germany, Italy, Japan, the U.K., and the U.S.) and eight developing countries in Asia and Latin America (Brazil, Chile, Indonesia, Korea, Malaysia, Mexico, Philippines, and Singapore). All data are taken from International Monetary Fund's *IFS* database. Unless indicated otherwise, quarterly data covering the sample period from 1974 Q1 to 2001 Q4 are analyzed. Shorter sample series are used in a few cases because of limited data availability. The specific sample periods for individual countries are described in Tables 1 and 2.

Real interest rates are computed as short-term nominal interest rates minus inflation rates. Three-month Treasury bill rates are used in the cases of Australia, Canada, France, Italy, Malaysia, Philippines, Singapore, the U.K., and the U.S. Twelve-month Treasury bill rates are used for Germany. One-month Treasury bill rates are used for Mexico. Three-month corporate bill rates are used for Korea. For the four other countries with no Treasury bill data, time deposit rates are examined: one-month deposit rates for Brazil and Chile, three-month deposit rates for Japan, and six-month deposit rates for Indonesia.

Real consumption is computed by dividing consumer spending by the consumer price level for the respective country. Quarterly consumption growth rates are analyzed for industrial countries. Due to limited availability of consumption data for developing countries, annual data are studied in those cases.

4. Some preliminary results

The augmented Dickey–Fuller test is often used for unit-root testing in previous studies. Despite being widely applied, this standard unit-root test is not efficient and has low power to uncover stationarity even when the series is actually stationary. The problem is addressed here using an efficient test. Elliott et al. (1996) derive the power envelope for various unit-root tests

Table 1
Results of the DF–GLS unit-root test on real consumption growth rates

	Sample period	p	DF–GLS statistic
Industrial countries			
Australia	1974 Q1–2001 Q4	1	−10.697**
Canada	1974 Q1–2001 Q4	2	−3.979**
France	1974 Q1–2001 Q4	0	−10.141**
Germany	1974 Q1–2001 Q4	0	−10.516**
Italy	1977 Q1–2001 Q4	0	−8.106**
Japan	1974 Q1–2001 Q4	0	−8.112**
U.K.	1974 Q1–2001 Q4	2	−2.918**
U.S.	1974 Q1–2001 Q4	2	−3.990**
Developing countries			
Brazil	1980–2001	0	−4.728**
Chile	1974–2001	0	−2.587*
Indonesia	1974–2001	0	−4.246**
Korea	1974–2001	0	−4.206**
Malaysia	1974–2001	0	−3.710**
Mexico	1974–2001	0	−3.547**
Philippines	1974–2001	3	−3.240**
Singapore	1974–2001	0	−3.551**

The real consumption growth rate, computed as $\Delta \ln(C_t)$, is examined. The lag parameter p is chosen based on the standard BIC. Critical values are estimated using the Monte Carlo method (Cheung and Lai, 1995), and each estimate is obtained based on 30,000 iterations. In the cases of the quarterly data for industrial countries, critical values for the DF–GLS test are estimated to be −2.09 and −2.69 for the 5% and 1% significance levels, respectively. In the cases of the annual data for developing countries, critical values are estimated to be −2.45 and −3.10 for the 5% and 1% significance levels. Statistical significance is indicated by an asterisk (*) for the 5% level and double asterisks (**) for the 1% level.

Table 2
Results of the DF–GLS unit-root test on real interest rates

	Sample period	p	DF–GLS statistic
Industrial countries			
Australia	1974 Q1–2001 Q4	3	−0.574
Canada	1974 Q1–2001 Q4	0	−1.773
France	1974 Q1–2001 Q4	2	−1.032
Germany	1975 Q3–2001 Q4	1	−3.294**
Italy	1977 Q1–2001 Q4	2	−0.795
Japan	1974 Q1–2001 Q4	3	−0.058
U.K.	1974 Q1–2001 Q4	3	−0.459
U.S.	1974 Q1–2001 Q4	2	−0.983
Developing countries			
Brazil	1982 Q4–2001 Q4	0	−5.546**
Chile	1977 Q1–2001 Q4	3	−0.038
Indonesia	1974 Q2–2001 Q4	4	−3.102**
Korea	1981 Q2–2001 Q4	3	−1.868
Malaysia	1974 Q1–2001 Q4	3	−1.242
Mexico	1978 Q1–2001 Q4	7	−2.001
Philippines	1976 Q1–2001 Q4	0	−5.585**
Singapore	1974 Q1–2001 Q4	0	−7.702**

The lag parameter p is chosen based on the standard BIC. Critical values for the DF–GLS test are −2.09 and −2.69 for the 5% and 1% significance levels, respectively. Statistical significance is indicated by an asterisk (*) for the 5% level and double asterisks (**) for the 1% level.

by analyzing the sequence of Neyman–Pearson tests of the unit-root null hypothesis ($\rho = 1$) against the local alternative of $\overline{\rho} = 1 + \overline{a}/T$ in an autoregressive model, for which $\bar{a} < 0$ is a constant and T is the sample size. Based on asymptotic power calculation, a modified Dickey–Fuller test, called the DF–GLS test, can achieve much higher statistical power than conventional unit-root tests. It is shown that efficient estimation of the largest root can be achieved by locally demeaning data through a generalized least squares (GLS) regression. Under the local alternative of $\rho < 1$, the GLS regression is carried out using the $(1 - \overline{\rho}L)$ transformation of the data, with L being the lag operator. The locally demeaned data are then evaluated using Dickey–Fuller-type tests. Let $\{y_t\}$ be the data process in general. The DF–GLS test entails the following regression:

$$(1 - L)\tilde{y}_t^{\mu} = \alpha\tilde{y}_{t-1}^{\mu} + \sum_{j=1}^{p} b_j(1 - L)\tilde{y}_{t-j}^{\mu} + u_t \tag{8}$$

where u_t is the error term and $\tilde{y}_t^{\mu}$, the locally demeaned series under the local alternative $\overline{\rho}$, is given by

$$\tilde{y}_t^{\mu} = y_t - z_t\widehat{\psi} \tag{9}$$

with z_t being a unit vector and $\widehat{\psi}$ being the simple regression coefficient of y_t^L on z_t^L, for which $y_t^L = (y_1, (1 - \overline{\rho}L)y_2, \ldots, (1 - \overline{\rho}L)y_T)'$ and $z_t^L = (z_1, (1 - \overline{\rho}L)z_2, \ldots, (1 - \overline{\rho}L)z_T)'$. The DF–GLS test statistic is given by the t-statistic testing for $\alpha = 0$ against the alternative of $\alpha < 0$ in Eq. (8). Elliott et al. (1996) recommend that the parameter $\bar{a}$, which defines the local alternative through $\overline{\rho} = 1 + \overline{a}/T$, be set equal to −7.

Table 1 reports the results of the DF–GLS test conducted on real consumption growth rates. The lag order p is selected using the standard Bayesian information criterion (BIC). The unit-root hypothesis is consistently rejected by the data for all the countries at the 5% significance level or better (strong evidence of stationarity can also be obtained when using the DF–GLS test that includes a time trend). The robustness of the rejection results across both industrial and developing countries fully supports Rose's (1988) observation that real consumption growth rates do not contain a unit root.

Table 2 contains the results of the DF–GLS test on real interest rates. These results are in sharp contrast to those for real consumption growth rates. Despite using an efficient unit-root test, it remains very hard to reject a unit root in the real interest rates for industrial countries: in only one of the eight cases can the unit-root hypothesis be rejected. Transformed series in terms of $\ln(1+r_t)$ were also tested, and they yielded qualitatively the same results as the real interest rate r_t series. For the developing countries, the results are mixed. Rejections of unit-root dynamics can be obtained in no more than half of the cases. Overall, the results from international data are far from supportive of no unit root.

Nearly all unit-root tests – including the DF–GLS test – take nonstationarity as the null hypothesis, which cannot be rejected unless there is very strong evidence against it. Arguably, such a setup tends to bias test results in favor of a unit root. It is therefore instructive to check whether the unit-root results for real interest rates are dependent on the specification of the null hypothesis. Taking a departure from the standard setup, Kwiatkowski et al. (1992) devise a Lagrange multiplier test, referred to as the KPSS test, under the null hypothesis of stationarity. Let e_t be the residual series obtained from a regression of y_t on a constant. The KPSS statistic, denoted by $\widehat{\eta}_\tau$, is constructed as

$$\widehat{\eta}_\tau = T^{-2}\sum_{t=1}^{T} S_t^2/s^2(l) \tag{10}$$

with

$$S_t = \sum_{t=1}^{l} e_t, \quad t = 1, 2, \ldots, T \tag{11}$$

$$s^2(l) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{j=1}^{l}[1 - j/(l+1)]\sum_{t=j+1}^{T} e_t e_{t-j} \tag{12}$$

where l is an autocorrelation lag truncation parameter and $s^2(l)$ is a heteroskedasticity and autocorrelation consistent variance estimator. Andrews (1991) suggests a data-dependent lag selection method based on optimal kernel bandwidth selection. The KPSS test is a one-sided upper tail test. When the test statistic is too large, the null hypothesis of no unit root will be rejected in favor of the alternative of a unit root.

Table 3 presents the results of the KPSS test performed on real interest rates. For the industrial countries, the null hypothesis of no unit root can be rejected in five of eight cases (Australia, Canada, Italy, Japan, and the U.K.). The results for the developing countries are equally unfavorable to the nonunit root hypothesis. The KPSS test still indicates the presence of a unit root in more than half the cases (Chile, Indonesia, Korea, Malaysia, and Mexico). In general, the KPSS results are not much different from the DF–GLS results. Neither the use of

Table 3
Results of the KPSS unit-root test on real interest rates

	l	KPSS statistic
Industrial countries		
Australia	7	0.621*
Canada	7	0.475*
France	13	0.373
Germany	19	0.143
Italy	4	0.513*
Japan	2	1.215**
U.K.	6	0.866**
U.S.	12	0.229
Developing countries		
Brazil	4	0.144
Chile	3	1.279**
Indonesia	8	0.464*
Korea	3	0.657*
Malaysia	4	0.754**
Mexico	4	0.491*
Philippines	6	0.229
Singapore	3	0.082

The autocorrelation lag truncation parameter l used in the KPSS test is chosen based on Andrews' (1991) optimal bandwidth selection method. The null hypothesis of no unit root is tested against the one-sided alternative of a unit root. Critical values for the KPSS test are 0.46 and 0.74 for the 5% and 1% significance levels, respectively (Kwiatkowski et al., 1992). Statistical significance is indicated by an asterisk (*) for the 5% level and double asterisks (**) for the 1% level.

an efficient unit-root test nor the alternative specification of the null hypothesis can produce significant evidence against a unit root in the real interest rate.

A remark is in order concerning the interpretation of the KPSS test results. The KPSS test tends to over-reject the nonunit root hypothesis when strong, albeit stationary, serial correlation exists. In addition, the KPSS test can find its root in the early type of partial sum test originally introduced in the statistic literature as a test for mean stability of a stationary time series (Gardner, 1969; MacNeill, 1978). A rejection with the KPSS test can thus arise from instability of the mean. In sum, when the KPSS test rejects, it may come from strong serial correlation or a unit root or from an unstable mean (the author owes this point to an anonymous referee). We next explore the mean-shift possibility.

5. A robust test for a mean shift at unknown time

To check for a possible mean shift in the real interest rate series, a new type of partial sum test proposed by Vogelsang (1998) is applied. This new test is attractive: it does not require estimation of serial correlation parameters, and the test is valid regardless of whether the time series is stationary or not. The test thus allows researchers to detect possible mean instability without worrying about serial correlation or stationarity of the data.

To outline the basic structure of Vogelsang's (1998) partial sum test, consider a time series process with a mean shift at time, say, $t = k$:

$$y_t = \mu + \delta \mathrm{DU}_t^k + w_t \tag{13}$$

where $\mathrm{DU}_t^k = I(t > k)$ with $I(\cdot)$ being the standard indicator function, and w_t is a general zero-mean error process. Using a simple data transformation based on partial sums, Eq. (13) can be rewritten as

$$s_t(y) = \mu t + \delta \mathrm{DT}_t^k + v_t \tag{14}$$

where $s_t(y) = \sum_{j=1}^{t} y_j$, $v_t = \sum_{i=1}^{t} w_i$ and $\mathrm{DT}_t^k = (t-k)I(t>k)$. Let $\mathrm{PS}_T(k)$ equal T^{-1} times the standard Wald statistic for testing $\delta = 0$ from least squares estimation of Eq. (14). Without prior knowledge of the actual shift date, $\mathrm{PS}_T(k)$ is computed over a range of possible shift dates, $\Lambda = \{t_m, t_m + 1, \ldots, T - t_m\}$, where t_m is the integer part of λT with λ often set equal to 0.1. Vogelsang (1998) shows that a mean exponential statistic of partial sums across all $k \in \Lambda$ can be constructed to test for a mean shift at an unknown date. The mean exponential statistic, which belongs to a class of optimal statistics proposed by Andrews and Ploberger (1994), is given by

$$\mathrm{Exp\,PS}_T = \ln\left\{T^{-1}\sum_{k\in\Lambda}\exp(\mathrm{PS}_T(k)/2)\right\}\exp\left(-bJ_T^*\right) \tag{15}$$

where b is a scaling parameter and $J_T^* = \inf_{k\in\Lambda} J_T(k)$ with $J_T(k)$ being equal to T^{-1} times the usual Wald statistic for testing $c_1 = c_2 = \cdots = c_9 = 0$ in the following least squares regression:

$$y_t = \mu + \delta \mathrm{DU}_t^k + \sum_{j=1}^{q} c_j t^j + e_t \tag{16}$$

where $q = 9$ is recommended based on local asymptotic analysis of size and power properties. As discussed by Vogelsang (1998), furthermore, the optimal choice of the scaling parameter, b, in Eq. (15) depends on the desired significance level:

$$b(P) = -8.986 + 42.543P - 60.427P^2 + 29.432P^3 + \exp(-99.324 + 100P) \tag{17}$$

where P is the percentile corresponding to the significance level used. $1 - P$ will give the asymptotic probability value (or p value) of the test. The J_T^* adjustment to the partial sum statistic serves essentially to correct the potential size distortion caused by persistent errors. Under the optimal choice of the scaling parameter, the asymptotic critical value of the $\mathrm{Exp\,PS}_T$ test statistic is the same whether the error process has a unit root or not.

The results of the $\mathrm{Exp\,PS}_T$ test on real interest rates are summarized in Table 4. A large majority of the cases indicates strong evidence of a mean shift in the real interest rate. For the industrial countries, the null hypothesis of no mean shift can be rejected at 5% significance level or better in six of eight cases (Australia, Canada, France, Italy, Japan, and the U.K.). For the developing countries, significant evidence against the no-shift null hypothesis can also be found in six of eight cases (Chile, Indonesia, Korea, Malaysia, Mexico, and Philippines). Interestingly, in all the earlier reported cases in which the KPSS test rejects, the $\mathrm{Exp\,PS}_T$ test rejects as well. Given the sensitivity of the KPSS test noted earlier, the rejection results from the KPSS test may likely reflect the statistical impact of an unstable mean.

6. Unit-root tests with mean-shift alternatives

In view of the substantial evidence of instability of the process mean, the unit-root hypothesis should be reevaluated with proper allowance for a mean shift when analyzing real interest

Table 4
Results of the mean exponential partial sum test for a mean shift

	Exp PS_T statistic (5% test)	Exp PS_T statistic (1% test)
Industrial countries		
Australia	5.976*	5.687**
Canada	2.612*	2.475
France	3.988*	3.699
Germany	0.264	0.249
Italy	2.693*	2.511
Japan	9.988*	9.710**
U.K.	16.343*	16.105**
U.S.	0.626	0.588
Developing countries		
Brazil	0.118	0.116
Chile	14.627*	14.257**
Indonesia	3.880*	3.850
Korea	5.823*	5.650**
Malaysia	7.625*	7.480**
Mexico	2.609*	2.582
Philippines	2.678*	2.645
Singapore	0.170	0.165

The null hypothesis of no mean shift is tested against the alternative of a mean shift. The Exp PS_T test is a one-sided upper tail test. Its test statistic contains a choice parameter that depends on the desired significance level under optimality conditions. Critical values for the mean-shift test are 2.03 and 4.63 for the 5% and 1% significance levels, respectively (Vogelsang, 1998). Statistical significance is indicated by an asterisk (*) for the 5% level and double asterisks (**) for the 1% level.

rate dynamics. To permit a mean shift in testing for a unit root, a variety of sequential tests will be performed. These tests extend Dickey–Fuller unit-root tests by accounting for a possible mean shift with no prior knowledge of the exact shift date. Treating the potential break date as unknown is desirable since the timing of the break, if any, can vary across data series and since any arbitrarily fixed date can be subject to criticism of data mining. The problem can be addressed by estimating the likely break date directly from the data using a sequential testing algorithm.

6.1. AO versus IO models

Two approaches for modeling structural breaks in time series have been advanced in the literature (Banerjee et al., 1992; Perron and Vogelsang, 1992; Vogelsang and Perron, 1998; Zivot and Andrews, 1992). One is the additive outlier (AO) approach that views the break as happening instantly, and the other is the innovational outlier (IO) approach that allows the break to take place gradually over time. Either approach can apply to both trending and nontrending data. Since including a deterministic time trend in real interest rate dynamics may raise theoretical issues, the following statistical analysis will focus on mean-shift models with no time trend.

Under the AO approach, the process of a mean shift is – as considered by Vogelsang (1998) – described by Eq. (13) with the innovation process w_t being generated by $(1-\rho L)A(L)w_t = B(L)v_t$, where $A(L)$ and $B(L)$ are lag polynomials with stable roots and v_t is white noise. When $\rho = 1$, y_t has a unit root. This model permits a mean shift to occur at time $t = k$.

To test for a unit root in the AO model, a demeaned series $\tilde{y}_t^a$ is first obtained as follows:

$$y_t = \mu + \eta \mathrm{DU}_t^k + \tilde{y}_t^a \tag{18}$$

where $\tilde{y}_t^a$ represents the least squares residual series. Next, a test for $\alpha_{\mathrm{AO}} = 0$ under the null hypothesis of a unit root is performed using the following regression:

$$(1-L)\tilde{y}_t^a = \sum_{j=0}^{p} \omega_j D_{t-j}(k) + \alpha_{\mathrm{AO}} \tilde{y}_{t-1}^a + \sum_{j=1}^{p} \phi_j (1-L)\tilde{y}_{t-j}^a + u_t \tag{19}$$

where u_t is the error term. The one-time dummy variables, $D_{t-j}(k) = I(t = k+j+1)$ for $j = 0, \ldots, p$, are included to ensure the test robustness with respect to the error correlation structure (see Perron and Vogelsang, 1992 for further discussion). For given values of k and p, the t-statistic for testing $\alpha_{\mathrm{AO}} = 0$ against the alternative of $\alpha_{\mathrm{AO}} < 1$ in regression (19) is denoted by $\tau_{\mathrm{DF}}(\mathrm{AO}, k, p)$.

To implement the above two-step procedure, the break point needs to be estimated from the data. By varying k in the regression over the range of possible shift dates Λ, the estimated break date, denoted by k_{B}, is obtained by minimizing over a sequence of t-statistics, $\tau_{\mathrm{DF}}(\mathrm{AO}, k, p)$ for all $k \in \Lambda$. This method has performed well in locating the true break point (Banerjee et al., 1992; Perron and Vogelsang, 1992; Zivot and Andrews, 1992). The unit-root test statistic is given by $\tau_{\mathrm{DF}}(\mathrm{AO}, k_{\mathrm{B}}, p) = \inf_{k \in \Lambda} \tau_{\mathrm{DF}}(\mathrm{AO}, k, p)$.

In contrast to the AO model, the IO model entertains situations in which the break may occur gradually over time. The IO model is particularly relevant when structural changes occur slowly, not instantly. The IO mean-shift model is specified as

$$y_t = \mu + \varphi(L)\left(\eta \mathrm{DU}_t^k + v_t\right) \tag{20}$$

where v_t is the innovation and $\varphi(L)$ is a polynomial function in lags, through which the lingering effects of a gradual structural break are captured. The long-run impact of a mean shift equals $\varphi(1)\eta$.

Following Perron and Vogelsang (1992), the following regression is estimated:

$$(1-L)y_t = \mu + \omega D_t(k) + \eta \mathrm{DU}_t^k + \alpha_{\mathrm{IO}} y_{t-1} + \sum_{j=1}^{p} \psi_j (1-L) y_{t-j} + \varepsilon_t \tag{21}$$

where ε_t is the error term. For given values of k and p, let $\tau_{\mathrm{DF}}(\mathrm{IO}, k, p)$ denotes the t-statistic for testing $\alpha_{\mathrm{IO}} = 0$ in regression (21). The break point k_{B} is chosen such that $\tau_{\mathrm{DF}}(\mathrm{IO}, k, p)$ is minimized over $k \in \Lambda$. The unit-root test statistic is computed as $\tau_{\mathrm{DF}}(\mathrm{IO}, k_{\mathrm{B}}, p) = \inf_{k \in \Lambda} \tau_{\mathrm{DF}}(\mathrm{IO}, k, p)$.

6.2. The GLS approach

A new study by Perron and Rodriguez (2003) shows that the efficient GLS approach to unit-root testing, as developed by Elliott et al. (1996), can be straightforwardly extended to structural-break models. In addition to deriving some asymptotic results for cases of a trend/level shift, the study uses simulation exercises to illustrate that GLS detrending can aid unit-root tests with trend-break alternatives in gaining significant test power.

Perron and Rodriguez's (2003) analysis makes no explicit distinction between AO and IO processes. One of the GLS-based tests considered by these authors involves a simple modification of the DF–GLS test. In our mean-shift case, the set of deterministic components of time series y_t is represented by $z_t^k = (1, \mathrm{DU}_t^k)'$. Let $y_t^L = (y_1, (1-\overline{\rho}L)y_2, \ldots, (1-\overline{\rho}L)y_T)'$ and $z_t^L = (z_1^k, (1-\overline{\rho}L)z_2^k, \ldots, (1-\overline{\rho}L)z_T^k)'$ be the respective locally demeaned series of y_t and z_t^k with the local alternative defined by $\overline{\rho} = 1 - 7/T$. The GLS unit-root test uses the following regression:

$$(1-L)\tilde{y}_t^g = \alpha_{\mathrm{GLS}}\tilde{y}_{t-1}^g + \sum_{j=1}^{p}\pi_j(1-L)\tilde{y}_{t-j}^g + u_t \qquad (22)$$

where $\tilde{y}_t^g = y_t - z_t^k\widehat{\zeta}$ with $\widehat{\zeta}$ being the least squares regression coefficient of y_t^L on z_t^L. For a known value of k, the standard Wald statistic for testing $\alpha_{\mathrm{GLS}} = 0$ in regression (22) is denoted by τ_{DF}(GLS, k, p) and has the same asymptotic distribution as the ADF test in a model with no mean (relevant critical values are tabulated by Hamilton, 1994). In the case of an unknown shift date, the break point k_{B} is chosen such that τ_{DF}(IO, k, p) is minimized over $k \in \Lambda$. The GLS test statistic is then given by $\tau_{\mathrm{DF}}(\mathrm{GLS}, k_{\mathrm{B}}, p) = \inf_{k\in\Lambda}\tau_{\mathrm{DF}}(\mathrm{GLS}, k, p)$.

6.3. Sequential unit-root test results

Each series of real interest rates is tested for a unit root using the various sequential tests. When performing each of these tests, the lag parameter p is chosen using the BIC. Table 5 summarizes the test results. Based on the AO test statistic, the unit-root null hypothesis can be rejected at the 5% significance level or better in seven of the eight cases of industrial countries (Australia, Canada, France, Germany, Italy, Japan, and the U.K.) and also in seven of the eight cases of developing countries (Brazil, Chile, Indonesia, Korea, Mexico, Philippines, and Singapore). The IO test statistic, by comparison, yields only slightly different results. Significant evidence against a unit root can be found in five cases of industrial countries (Canada, France, Germany, Japan, and the U.K.) and in all eight cases of developing countries.

Compared to both AO and IO statistics, the GLS test statistic produces even broader and more consistent evidence against the unit-root hypothesis. When a mean shift is admitted under the alternative hypothesis, the unit-root null can be rejected in all but one of the cases of industrial countries and in all the cases of developing countries. All in all, the allowance for a mean shift helps find stronger and broader evidence against a unit root than our earlier analysis.

In summary, there is broad international evidence that real interest rates do not contain a unit root. The findings here underscore an important point that the existence of a structural break, if it is not properly accounted for, can induce tests to spuriously find unit-root nonstationarity, even though the real interest rate actually contains no unit root.

7. Further analysis: structural break or long memory

Empirical evidence of long memory has been reported in some studies of real interest rate dynamics for the U.S. (e.g., Lai, 1997b; Tsay, 2000). Unit-root tests generally have low power against long memory alternatives of slow mean reversion. A number of studies (Cheung, 1993; Parke, 1999; Granger, 2000; Diebold and Inoue, 2001) show that tests for long memory can pick up nonlinear dynamics, including a mean shift. Accordingly, the mean-shift results here may be viewed as complementary to the evidence of long memory. In either case, rejections

Table 5
Results of sequential unit-root tests with mean-shift alternatives

	p	τ_{DF}(AO, k_B, p)	p	τ_{DF}(IO, k_B, p)	p	τ_{DF}(GLS, k_B, p)
Industrial countries						
Australia	3	−4.959*	3	−3.312	0	−4.616**
Canada	0	−6.638**	0	−6.588**	0	−5.744**
France	0	−5.935**	0	−5.884**	0	−5.506**
Germany	1	−4.648**	1	−4.641*	1	−4.292**
Italy	0	−6.152**	2	−4.048	0	−5.531**
Japan	4	−4.931*	3	−5.683**	6	−3.703*
U.K.	0	−9.787**	4	−7.056**	4	−4.599**
U.S.	2	−3.727	2	−3.921	2	−2.901
Developing countries						
Brazil	0	−13.175**	0	−13.274**	0	−5.820**
Chile	0	−11.476**	0	−11.356**	0	−9.776**
Indonesia	3	−6.672**	3	−6.704**	3	−6.383**
Korea	0	−9.393**	0	−9.477**	0	−6.889**
Malaysia	3	−4.464	0	−9.288**	0	−8.940**
Mexico	0	−8.737**	0	−8.646**	0	−7.707**
Philippines	0	−7.928**	2	−7.716**	0	−6.328**
Singapore	0	−8.576**	0	−8.506**	0	−8.214**

One-sided lower tail tests are considered here. The unit-root null hypothesis is tested against the one-sided alternative of no unit root but a mean shift. The lag parameter p is chosen using the BIC for every test. For the AO and IO models, critical values are tabulated by Perron and Vogelsang (1992). For the AO model, critical values are, respectively, −4.41 and −5.05 for the 5% and 1%. For the IO model, critical values are −4.39 and −5.03 for the 5% and 1% levels, respectively. In the case of the GLS test, critical values are obtained using the simulation method based on 30,000 iterations, and they are −3.49 and −4.06 for the 5% and 1% levels, respectively. Statistical significance is indicated by an asterisk (*) for the 5% level and double asterisks (**) for the 1% level.

of the unit-root null are obtained from permitting some nonlinearity under the alternative hypothesis.

The question then is whether long-memory models can be as good as mean-shift models in explaining the international data on real interest rates. The long-memory property of a time series process is governed by the fractional integration order (denoted by d) of the process. When $d = 1$, the process has a unit root and is not mean reverting. For $0 < d < 1$, the process exhibits long memory with a slow reversion rate. Standard unit-root tests consider merely $d = 0$ under the alternative hypothesis, not allowing for the long-memory possibility of $0 < d < 1$. To evaluate the relevance of the long-memory explanation, the spectral GPH test (Geweke and Porter-Hudak, 1983) for long memory is carried out. This semi-nonparametric test is attractive for not requiring any explicit parameterization of the serial correlation structure. Let x_t be the differenced r_t series (i.e., $x_t = \Delta r_t$). In the following spectral regression, one plus the slope coefficient ϕ will give the GPH estimate of the long memory parameter d:

$$\ln\left(I_T\left(\omega_j\right)\right) = c - \phi \ln\left(4\sin^2\left(\omega_j/2\right)\right) + u_t, \quad j = 1, 2, \ldots, m \tag{23}$$

where $I_T(\omega_j)$ is the periodogram at the jth Fourier frequency $\omega_j = 2\pi j/T$ such that

$$I_T\left(\omega_j\right) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t\, e^{i\omega_j t}\right|^2 \tag{24}$$

and $m = T^{\upsilon}$ for $0 < \upsilon < 1$. Computationally, the periodogram can be obtained as follows:

$$I_T(\omega_j) = \frac{1}{2\pi T}\left[\left(\sum_{t=1}^{T} x_t \cos(\omega_j t)\right)^2 + \left(\sum_{t=1}^{T} x_t \sin(\omega_j t)\right)^2\right] \tag{25}$$

The theoretical error variance for the ϕ estimate is known to be equal to $\pi^2/6$. A unit-root test can be performed based on the t-statistic for ϕ. The null hypothesis of $d = 1$ (i.e., a unit root) is tested against the mean-reverting long-memory alternative of $d < 1$. If the ϕ estimate is significantly less than zero, then the unit-root null is rejected. It should be noted that the number of low-frequency ordinates, indicated by m, used in the spectral regression is a choice variable. If m is too large, the estimation may be biased due to contamination caused by high-frequency dynamics. If m is too small, it will produce imprecise estimates due to a limited degree of freedom for estimation. To balance these two consideration factors, $\upsilon = 0.50$ is used for the sample-size function, $m = T^{\upsilon}$. In fact, use of $m = T^{0.5}$ has emerged as a popular rule of thumb in the empirical literature.

Table 6 gives the results of the GPH test on real interest rates. Evidently, long-memory models produce little evidence against the unit-root hypothesis for industrial industries, although they perform reasonably well in rejecting a unit root for developing countries. The unit-root null can be rejected in merely one case of industrial countries (Germany) and in six cases of developing countries (Brazil, Indonesia, Malaysia, Mexico, Philippines, and Singapore). Overall, long-memory models fail to fully explain the real interest rate dynamics in our cross-country data.

Table 6
Results of the GPH test for long memory

	GPH statistic
Industrial countries	
Australia	−0.861
Canada	−0.772
France	−1.063
Germany	−1.759*
Italy	−1.171
Japan	−0.439
U.K.	−0.559
U.S.	−0.834
Developing countries	
Brazil	−2.448*
Chile	−0.359
Indonesia	−3.732**
Korea	−0.943
Malaysia	−2.036*
Mexico	−1.730*
Philippines	−4.381**
Singapore	−2.292*

The null hypothesis of a unit root ($d = 1$) is tested against the one-sided long-memory alternative of mean reversion ($d < 1$). Critical values are estimated using the Monte Carlo method based on 30,000 iterations, and they are estimated to be −1.71 and −2.63 for the 5% and 1% significance levels, respectively. Statistical significance is indicated by an asterisk (*) for the 5% level and double asterisks (**) for the 1% level.

8. Conclusion

Previous studies, which focus mainly on industrial countries, generally have great difficulty in rejecting unit-root dynamics for real interest rates. This study shows that the apparent unit-root behavior can arise from the common failure to properly account for possible structural breaks. Structural breaks, when they occur, can be misinterpreted by unit-root tests as permanent shocks, thereby undermining the ability of these tests to detect long-run reversion. This study reexamines the relevance of the Fisher effect and analyzes international data for both industrial and developing countries. Without allowing for any structural breaks, unit-root tests often fail to detect stationarity in real interest rates, especially for industrial countries. However, when unit-root tests permitting a mean shift are applied, strong evidence in favor of no unit root can be uncovered, rejecting unit-root dynamics for both industrial and developing countries. It follows that real interest rates may appear nonstationary when in fact they are stationary and structural breaks are responsible. Other possible explanations – including the inefficiency of testing procedures and the presence of long-memory dynamics – have also been explored, but only the structural-break explanation can adequately account for the widespread failure to reject unit-root dynamics. In finding no unit root in real interest rates, the new results resolve the puzzling inconsistency with intertemporal consumption behavior, as implied by consumption-based asset pricing models. The nonunit root results from the cross-country data also lend wide support to the Fisher effect.

Finally, it should be noted that the empirical results in this study by no means suggest the existence of exactly one structural break in individual real interest rate processes. Indeed, the study makes no attempt to estimate the number of possible structural breaks. The actual number of breaks may be more than one, and multiple structural breaks may occur for some data series. Nonetheless, it would not alter the main finding, namely, the structural-break possibility can explain the widespread failure in rejecting the unit-root hypothesis. The allowance of just one mean shift is enough to uncover strong international evidence that real interest rates generally contain no unit root, as predicted by the long-run Fisher condition.

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