

# Long memory and nonlinear mean reversion in Japanese yen-based real exchange rates

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## Abstract

The extraordinary difficulty in uncovering parity reversion in yen-based real exchange rates has often been ascribed to a missing trend variable. This study identifies an alternative explanation and shows that the puzzling behavior of real yen rates may stem from long-memory dynamics, which undermine unit-root tests in their ability to detect mean reversion. The long-memory findings are consistent with the long swings in yen exchange rates during the current float. Further analysis also reveals evidence of non-monotonic reversion toward parity. © 2001 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

Although significant short-run departures from purchasing power parity (PPP) have been widely reported, many economists maintain the view that PPP will prevail in the long run. The recent float experience has not always been reassuring, however, weakening belief and inducing shifts in sentiment. Early studies in the 1980s commonly failed to find parity reversion in real exchange rates during the post-Bretton Woods period (see the extensive reviews by Froot and Rogoff, 1995; Rogoff, 1996, and the studies cited therein). The widespread failure to detect parity reversion seri-

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ously challenged the faith in PPP. Only until the 1990s did many studies begin to unveil supportive evidence of reverting dynamics under the current float (Abuaf and Jorion, 1990; Frankel and Rose, 1996; Lothian, 1990; Lothian and Taylor, 1996; Oh, 1996; Papell, 1997; Sarno and Taylor, 1998; Taylor and Sarno, 1998; Wu, 1996).

The apparent revival of the empirical relevance of PPP has been typically attributed to the improved statistical power attained through using efficient testing procedures and/or longer sample series available for the current float (Lothian and Taylor, 1997). This generic explanation in terms of purely statistical power, however, says little about one of the stylized facts of floating exchange rates, namely, that it is notably much harder to detect PPP reversion when particular currencies — such as the US dollar and the Japanese yen — are employed as the numeraire currency (Jorion and Sweeney, 1996; Koedijk et al., 1998; Papell, 1997; Papell and Theodoridis, 1998a,b; Wei and Parsley, 1995).

This study analyzes specifically the dynamics of yen-based real exchange rates during the current float. Researchers have been confronted with comparable, if not greater, difficulty in detecting PPP reversion in real yen rates as opposed to real dollar rates (Cheung and Lai, 1998; Koedijk et al., 1998; Papell and Theodoridis, 1998b), except for long historical data (Lothian, 1990). Although the difficulty in uncovering PPP may be partly reduced by including a linear trend variable, it remains generally hard to find strong evidence of mean reversion among yen-based real exchange rates. The inclusion of a time trend seems at variance with the standard version of long-run PPP. It has, however, been rationalized as capturing the Balassa–Samuelson effect of productivity growth.

A missing trend variable, this study shows, may not be the key explanation for the behavior of yen-based real exchange rates over the current float. There are more intriguing and pertinent dynamics that are responsible for the usual difficulty in detecting PPP reversion in real yen rates. These are long-memory dynamics, and they can confound unit-root tests and undermine their ability to distinguish between the high-frequency and low-frequency dynamics. This study demonstrates that when the long-memory dynamics are properly accounted for in statistical tests based on fractional time series models, strong evidence of mean reversion can be uncovered in real yen rates.

The finding of long-memory dynamics may be symptomatic of long-swing dynamics. Yen exchange rates are buffeted by notably large appreciations and depreciations over an extended period of time during both the 1980s and the 1990s (see Fig. 1). Lothian (1998) points out the potential implications of long currency swings for PPP analysis. In studying closely the behavior of dollar exchange rates, the author observes that there can be more behind the story of PPP re-emergence. The exceptional difficulty in uncovering parity reversion may be ascribable to specific US economic events. The substantial dollar appreciation and depreciation between 1980 and 1987, in particular, make it unusually hard for researchers to separate statistically the long-run from the short-run dynamics and identify mean reversion (the long swings in the dollar have earlier been noted by Engle and Hamilton, 1990). Given the limited time span of the current float data, the impact of the long swings on statistical tests can be especially significant. This poses an important challenge for

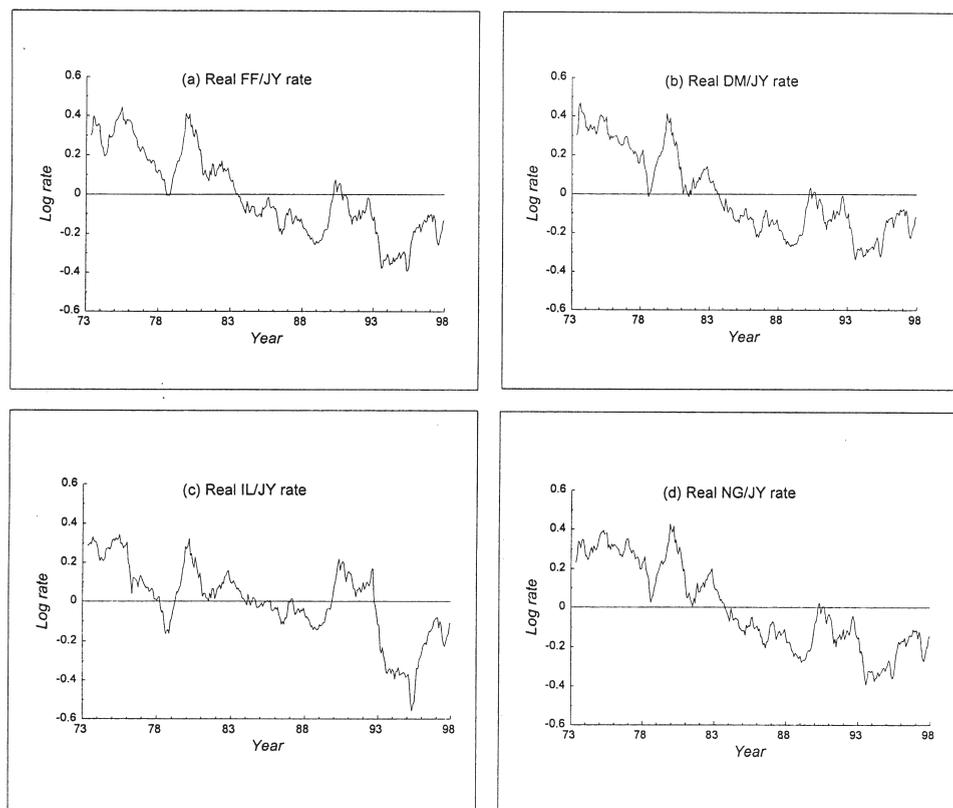


Fig. 1. Plots of yen-based real exchange rates.

conventional econometric analysis. Lothian's (1998) study highlights a potential deficiency of standard time series models in handling long-swing dynamics. It calls for better empirical modeling of the long-swing behavior of real exchange rates so that the high- and low-frequency components can be distinguished effectively.

We observe that the data problem caused by long-swing dynamics may similarly afflict yen exchange rates. This problem will be tackled using a highly flexible time series model, which can capture a vast range of high- and low-frequency dynamics at the same time. Specifically, the real exchange rate behavior will be modeled by a class of generalized univariate processes, called fractionally integrated processes (Granger and Joyeux, 1980; Hosking, 1981). Fractional dynamics — related to non-linear dynamics called “fractal” (Mandelbrot, 1977) — are known to be characterized by irregular long cycles and long-term memory. Since fractionally integrated processes are flexible enough to describe both large swings and mean-reverting dynamics simultaneously, they readily lend themselves to model the real exchange rate behavior. In fact, fractional models are able to capture subtle mean reversion because they offer better approximation for the low-frequency dynamics than standard time

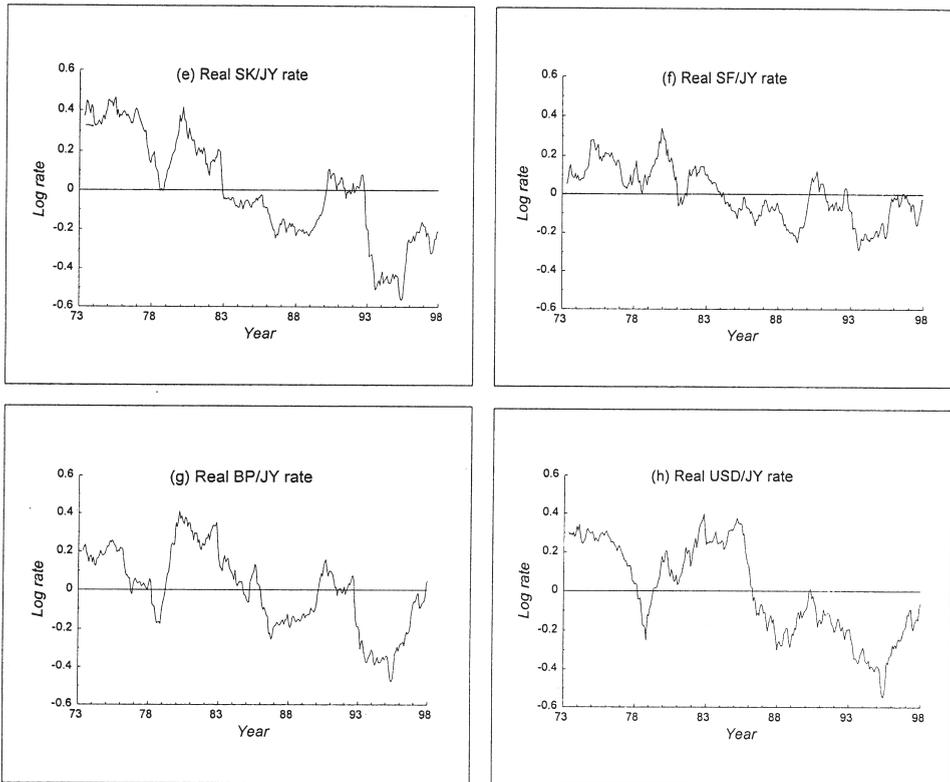


Fig. 1. (continued)

series models. The use of fractional models for exchange rate dynamics is not too uncommon. Cheung and Lai (1993) and Diebold et al. (1991) find mean-reverting long-memory dynamics in long historical series of real exchange rates.

To inquire further into the dynamic process of real exchange rate adjustment, impulse response analysis based on the fractional model is conducted. Estimating impulse responses permits researchers to gain detailed information on how the real exchange rate adjusts over both short and long horizons in response to a shock. Taylor et al. (1999) recently explore potential nonlinearity in PPP reversion for real dollar rates. In the study here, evidence of nonlinearity in the form of non-unidirectional mean reversion is uncovered from real yen rates. The real yen rate tends to overreact to shocks and magnify the initial shock impact such that the rate moves further away from parity before reverting. Interestingly, similar short-term overreacting behavior has been found in real dollar rates (Cheung and Lai, 2001).

## 2. Empirical modeling using fractional processes

The possibility of long swings as an intrinsic characteristic of exchange rate dynamics has long been recognized. Kaen and Rosenman (1986) note that speculative markets — including those of currencies and securities — are prone to sharp swings and irregular cycles. Under imperfect information and bounded rationality, investors are likely to follow some rule-governed behavior. Such behavior can produce persistent price movements in the same direction, with their subsequent abrupt reversals. The Kaen–Rosenman model has been used to explain the presence of long swings in many financial prices. Long swings may also be consistent with Frankel and Froot's (1990, 1993) view of exchange rate dynamics. By distinguishing between chartists (traders who like to follow recent trends and tend to have bandwagon expectations) and fundamentalists (traders who base their forecasts on economic fundamentals and tend to have regressive expectations), it is suggested that the overvaluation of the dollar over the 1981–85 period might be the result of market forecasts being dominated by chartists. According to these authors, the dollar in 1985 “overshot the overshooting equilibrium”, setting the stage for the great depreciation in 1986.

Empirically, the presence of long-swing dynamics has much bearing on statistical modeling and testing. Lothian's (1998) observation about the real exchange rate behavior underscores a basic problem in analyzing mean-reverting behavior. A test of mean reversion entails proper modeling of the low-frequency dynamics, while allowing for possibly persistent dynamics in the short run. It follows that empirical results can depend critically upon the ability of the applied statistical technique to separate the low-frequency (permanent) from the high-frequency (transitory) components. The presence of long swings blurs and muddles the distinction between the different dynamic components, making it extraordinarily difficult to identify mean reversion. Proper allowance for long-swing dynamics thus becomes pivotal for PPP testing.

Irregular long-cycle dynamics are embodied as part of the inherent behavior of fractionally integrated processes (Mandelbrot, 1972). These processes constitute a general class of flexible time series models (Granger and Joyeux, 1980; Hosking, 1981), described by

$$B(L)(1-L)^d y_t = D(L) \varepsilon_t, \quad (1)$$

where  $L$  is the lag operator;  $B(L) = 1 - \beta_1 L - \dots - \beta_p L^p$  and  $D(L) = 1 + \delta_1 L + \dots + \delta_q L^q$  are polynomials with stable roots;  $\varepsilon_t$  is white noise;  $d$  is the fractional order parameter; and the fractional differencing operator,  $(1-L)^d$ , yields an infinite-order lag polynomial with slowly declining coefficients as follows:

$$(1-L)^d = \sum_{r=0}^{\infty} \Gamma(r-d) L^r / \{\Gamma(r+1)\Gamma(-d)\}, \quad (2)$$

where  $\Gamma(\cdot)$  is the gamma function. Due to the presence of significant dependence between distant observations, fractional processes are called long-memory processes.

Model (1) extends conventional autoregressive and moving average (ARMA) models to non-integer values of  $d$ . Unit-root processes are included as a special case with  $d=1$ ; whereas, stationary ARMA processes represent another special case with  $d=0$ . Without limiting the value of  $d$  to either zero or unity, the fractional process allows for a very rich class of spectral behavior at low frequencies. Its spectral density,  $f_y(\lambda)$ , behaves like  $\lambda^{-2d}$  as  $\lambda \rightarrow 0$ . For  $d > 0$ ,  $f_y(\lambda)$  is unbounded at frequencies approaching zero, rather than bounded as for stationary ARMA processes. By permitting  $d$  to take non-integer values, the fractional model accommodates a broader range of low-frequency, mean-reverting dynamics than standard time series models.

The mean-reverting property hinges critically upon whether  $d < 1$ . The impact of a shock is known to persist forever for a unit-root ( $d=1$ ) process. A fractional process with  $d < 1$ , on the other hand, exhibits shock dissipation. This can be seen from the moving average representation for  $(1-L)y_t$ :

$$(1-L)y_t = A(L)\varepsilon_t, \quad (3)$$

where  $A(L) = 1 + \alpha_1 L + \alpha_2 L^2 + \alpha_3 L^3 + \dots$ , derived from

$$A(L) = (1-L)^{1-d} \Phi(L) \quad (4)$$

for  $\Phi(L) = B^{-1}(L)D(L)$ . The moving average coefficients  $\{\alpha_1, \alpha_2, \alpha_3, \dots\}$  are referred to as the impulse responses. The cumulative impulse response over  $j$  periods of time is given by

$$C_j = 1 + \alpha_1 + \alpha_2 + \dots + \alpha_j \quad (5)$$

and it tracks the impact of a unit innovation at time  $t$  on the real exchange rate at time  $t+j$ . As  $j \rightarrow \infty$ ,  $C_\infty = A(1)$ , measuring the long-run impact of the innovation (Campbell and Mankiw, 1987). Cheung and Lai (1993) show that for  $d < 1$ ,  $C_\infty = 0$ , implying shock-dissipating behavior. For  $d \geq 1$ ,  $C_\infty \neq 0$  and so the effect of a shock will not die out. Mean reversion (i.e.,  $C_\infty = 0$ ) occurs so long as  $d < 1$ . It follows that a test for fractional integration can be used to determine the existence of mean reversion.

### 3. Data and empirical results

Real exchange rates for the Japanese yen (JY) vis à vis the US dollar (USD) and seven major European currencies — including the French franc (FF), the German mark (DM), the Italian lira (IL), the Dutch guilder (NG), the Swedish krona (SK), the Swiss franc (SF) and the British pound (BP) — are examined. The data series under study are monthly real rates constructed from consumer price indices and nominal exchange rates. Taken from the International Monetary Fund's *International Financial Statistics* CD-ROM, the data cover the sample period from April 1973 through December 1997. Following the common practice in the PPP literature, all the series of real exchange rates are expressed in logarithms. Fig. 1 plots yen-based real exchange rates in eight bilateral cases. The charts indicate the general presence of large swings in real yen rates during both the 1980s and the 1990s. Underlying

the sizable swings, some patterns of reversals may be observed. Nonetheless, the statistical significance of the mean-reverting tendency has been hard to establish, especially with univariate tests. Confronting the empirical challenge, this study still uses univariate procedures. This permits us to obtain direct evidence of reversion for each individual series without dealing with any possible issues associated with multivariate procedures (Abuaf and Jorion, 1990; O’Connell, 1998; Papell, 1997; Sarno and Taylor, 1998; Taylor and Sarno, 1998).

To serve as a benchmark for comparison, all series of real exchange rates are first tested for a unit root using the augmented Dickey-Fuller (ADF) test. For PPP reversion to hold, the real exchange rate should be stationary and contain no unit root. The ADF test entails the following regression:

$$(1-L)y_t = \mu_0 + \mu_1 t + \zeta_0 y_{t-1} + \sum_{j=1}^p \zeta_j (1-L)y_{t-j} + \epsilon_t, \tag{6}$$

where  $L$  is the lag operator and  $\epsilon_t$  is the random error term. For the test without a time trend,  $\mu_1=0$ . The null hypothesis of a unit root is given by  $\zeta_0=0$ . The ADF statistic is the  $t$ -statistic for the  $\zeta_0$  coefficient.

Table 1 contains the empirical results from the ADF tests with and without a time trend. The lag parameter is chosen using a data-dependent method based on the usual Akaike information criterion (AIC). In the no-trend case, the ADF test produces little support for PPP reversion. In merely one (IL/JY) case can the null hypothesis of a unit root be rejected in favor of stationary alternatives at the 10% level of significance. Adding a time trend to the ADF test may help improve the test performance, but the results still fall short of being supportive of long-run PPP. In just three (FF/JY, IL/JY and SF/JY) out of the eight cases can the unit-root null be rejected

Table 1  
ADF tests on yen-based real exchange rates<sup>a</sup>

Exchange rate	ADF without trend		ADF with trend	
	Lag	Statistic	Lag	Statistic
FF/JY	1	-1.753	8	-3.479**
DM/JY	1	-1.809	1	-2.470
IL/JY	5	-2.705*	5	-3.443**
NG/JY	1	-1.514	2	-2.749
SK/JY	3	-1.955	3	-2.915
SF/JY	1	-2.192	7	-3.780**
BP/JY	1	-1.898	4	-2.477
USD/JY	1	-1.620	1	-1.818

<sup>a</sup> Eight currencies against Japanese yen (JY) are considered, and they are French franc (FF), German mark (DM), Italian lira (IL), Dutch guilder (NG), Swedish krona (SK), Swiss franc (SF), British pound (BP), and the US dollar (USD). Critical values for the unit-root tests are based on Cheung and Lai (1995). Statistical significance is indicated by a single asterisk (\*) for the 10% level and a double asterisk (\*\*) for the 5% level.

at the 5% significance level. The inclusion of a linear time trend, therefore, is insufficient for explaining the behavior of real yen exchange rates. How about a trend with a break? Investigating the possibility of structural instability may also be useful before identifying fractional dynamics (Cheung, 1993).

To explore the empirical relevance of the trend-break possibility, a number of sequential unit-root tests are performed. These sequential tests extend the ADF test by accounting for a possible jump or shift in trend in the data process, with no prior knowledge of the break date. The treatment of an unknown break date is desirable since the timing of the break, if any, can vary across series of real exchange rates and since any arbitrarily fixed date can be subject to criticism of data mining. Two basic methods of modeling trend breaks in time series have been considered in the literature (Banerjee et al., 1992; Vogelsang and Perron, 1998; Zivot and Andrews, 1992). One is the additive outlier (AO) approach that views the break as occurring instantly, and the other is the innovational outlier (IO) approach that allows the change to take place gradually over time. Three different versions of both AO and IO models are considered here, and their corresponding broken-trend tests with endogenous selection of the break point are described in Appendix A.

Each series of real exchange rates is tested for a unit root using various sequential tests under both AO and IO settings, without restricting a priori the analysis to any specific type of structural breaks (instant or gradual; mean or trend shift). In performing each of these tests, the lag parameter  $p$  is chosen using a data-dependent method based on the AIC, as used for the ADF test. Table 2 contains the results of sequential tests under different AO models. Evidently, the AO broken-trend models

Table 2  
Unit-root tests under additive outlier models of a broken trend<sup>a</sup>

Bilateral series	Mean-shift model		Trend-shift model		Combined model	
	$p$	$\tau_{DF}(AO, \bar{n}, a)$	$p$	$\tau_{DF}(AO, \bar{n}, b)$	$p$	$\tau_{DF}(AO, \bar{n}, c)$
FF/JY	1	-2.795	1	-2.994	1	-2.913
DM/JY	1	-3.146	1	-3.744	1	-3.865
IL/JY	1	-2.538	1	-2.349	1	-3.210
NG/Y	1	-3.412	1	-3.562	1	-3.627
SK/JY	1	-2.783	1	-2.661	1	-2.638
SF/JY	1	-3.630	1	-3.598	1	-3.630
BP/JY	1	-1.790	1	-1.960	1	-1.917
USD/JY	1	-2.474	1	-1.733	1	-2.469

<sup>a</sup> This table contains unit-root test results under different additive outlier models of a trend break. The lag parameter  $p$  for each test is selected using a data-dependent method based on the AIC, with the maximum lag order=10. The unit-root test statistics given by  $\tau_{DF}(AO, \bar{n}, i)$  for  $i=a, b$  and  $c$  are, respectively, obtained based on regressions (A7), (A8) and (A9). Asymptotic critical values for individual statistics are provided by Vogelsang and Perron (1998). For the mean-shift model, the critical values are given by -3.90 (the 10% significance level) and -4.17 (the 5% level). For the trend-shift model, the critical values are given by -4.04 (the 10% level) and -4.34 (the 5% level). For the combined model, the critical values are given by -4.31 (the 10% level) and -4.61 (the 5% level). All the test statistics reported above are not significant at the 10% level.

do not help explain the behavior of real yen rates. In no case can the null hypothesis of a unit root be rejected, regardless of whether a mean-shift or a trend shift or both is included under the alternative hypothesis. Table 3 gives the results of sequential tests under different IO models. Again, all broken-trend models produce no significant evidence against a unit root. In sum, the overall results suggest that the allowance for a broken time trend remains inadequate to account for the dynamics of real yen rates.

An alternative explanation for the real exchange rate behavior is explored next. The tendency to revert may have been obscured by the presence of long-memory dynamics in real yen rates. These long-memory dynamics, if they not properly accounted for, can foil statistical tests in their ability to detect stationarity. To unveil parity reversion, fractional models are applied to capture long-memory dynamics and identify stationarity. The fractional model, as described by model (1), is fitted to each individual series of real yen rates. Since the low-frequency behavior is parameterized by the fractional integration parameter,  $d$ , and mean reversion exists when  $d < 1$ , the latter condition provides the basis for a fractional test of parity reversion in the real exchange rate.

For statistical testing, the fractional integration parameter can be estimated using a frequency-domain maximum likelihood procedure. Following Fox and Taquq (1986), we exploit the property that maximization of the likelihood function is asymptotically equivalent to minimization of

Table 3  
Unit-root tests under innovational outlier models of a broken trend<sup>a</sup>

Bilateral series	Mean-shift model		Trend-shift model		Combined model	
	$p$	$\tau_{DF}(\text{IO}, \tilde{n}, a)$	$p$	$\tau_{DF}(\text{IO}, \tilde{n}, b)$	$p$	$\tau_{DF}(\text{IO}, \tilde{n}, c)$
FF/JY	1	-2.919	1	-3.030	1	-2.429
DM/JY	1	-3.684	1	-3.762	1	-4.040
IL/JY	1	-3.393	1	-2.402	1	-4.037
NG/JY	1	-3.626	1	-3.531	1	-3.947
SK/JY	1	-2.891	1	-2.697	1	-3.413
SF/JY	1	-3.656	1	-3.614	1	-3.861
BP/JY	1	-2.356	1	-2.202	1	-3.233
USD/JY	1	-3.018	1	-2.146	1	-2.829

<sup>a</sup> This table provides unit-root test results under different innovational outlier models of a trend break. The lag parameter  $p$  for each test is selected using a data-dependent method based on the AIC, with the maximum lag order=10. The unit-root test statistics given by  $\tau_{DF}(\text{IO}, \tilde{n}, j)$  for  $j=a, b$  and  $c$  are, respectively, obtained from regressions (A13), (A14) and (A15). Critical values for the crash model and the changing growth models are both based on Banerjee et al. (1992); those for the mean-shift model are given by -4.50 (the 10% significance level) and -4.79 (the 5% significance level) for  $T=250$ , and those for the trend-shift model are given by -4.12 (the 10% level) and -4.39 (the 5% level) for  $T=250$ . For the combined model, asymptotic critical values are available from Vogelsang and Perron (1998); they are given by -4.86 (the 10% level) and -5.16 (the 5% level). All the test statistics reported in this table are not significant at the 10% level.

$$\sum_{k=1}^{T-1} I_y(2\pi k/T) / f_y(2\pi k/T; \xi) \tag{7}$$

with respect to  $\xi=(d, \beta_1, \dots, \beta_p, \delta_1, \dots, \delta_q)$ , where  $I_y(\lambda)$  is the periodogram of  $y$  at frequency  $\lambda$ , and  $f_y(\lambda, \xi)=|1 - e^{-i\lambda}|^{-2d} |B^{-1}(e^{-i\lambda})D(e^{-i\lambda})|^2$  is proportional to the spectral density of  $y$  at frequency  $\lambda$ . The resulting estimator for  $d$  is consistent and has an asymptotic normal distribution.

Table 4 displays the results of fractional integration analysis. Maximum likelihood estimates of the integration order,  $d$ , are obtained using the Davidson–Fletcher–Powell algorithm and based on the model specifications selected by the AIC, with both  $p$  and  $q$  being permitted to be less than or equal to four in model (1). Similar parsimonious model restrictions on  $p$  and  $q$  have commonly been made in the empirical literature on fractional time series to reduce computation burden, given that low orders of  $p$  and  $q$  are normally found to be adequate for fractionally integrated models. In our case, an order of less than four for both  $p$  and  $q$  is enough to capture the dynamics of all the real exchange rate series under study.

The results reported in Table 4 indicate that all the real yen rate series have an integration order of neither zero nor unity. The data uniformly reject the null of  $d=0$  in favor of fractional alternatives of  $d>0$  at the 5% significance level or better. More importantly, the null of an exact unit root (i.e.,  $d=1$ ) can also be rejected in favor of mean-reverting alternatives of a fractional unit root ( $d<1$ ) at the 1% significance level in all the cases. These results strongly support the existence of mean-reverting fractional dynamics in individual series of real yen exchange rates.

To illustrate the potential impact of fractional dynamics on unit-root testing, a Monte Carlo experiment is conducted to assess the power of the ADF test against fractional dynamics. The data generating processes are based on the fractional model specifications estimated from actual data of the various real exchange rates. The ADF test with a time trend is applied to the simulated fractional series in each

Table 4  
Results from fractional integration analysis<sup>a</sup>

Bilateral series	$d$	Standard error	Testing $H_0: d=0$ versus $H_1: d>0$	Testing $H_0: d=1$ versus $H_1: d<1$
FF/JY	0.238	0.139	1.711**	-5.482***
DM/JY	0.166	0.077	2.155**	-10.858***
IL/JY	0.180	0.105	1.714**	-7.828***
NG/JY	0.138	0.066	2.072**	-12.972***
SK/JY	0.208	0.067	3.132***	-11.894***
SF/JY	0.121	0.071	1.706**	-12.432***
BP/JY	0.194	0.115	1.685**	-6.985***
USD/JY	0.126	0.041	3.073***	-21.382***

<sup>a</sup> The second column provides the estimates of the fractional integration parameter,  $d$ , corresponding to individual series of real exchange rates. The fourth and fifth columns give the respective  $t$ -statistics for hypothesis testing. Statistical significance is indicated by a double asterisk (\*\*) for the 5% level and a triple asterisk (\*\*\*) for the 1% level. See also Table 1 for other notes.

replication, and the percentage of rejections at the 5% significance level is then recorded from 10,000 samples of the artificial data for each real exchange rate process. The power estimates are obtained as follows: 26.7% (the French franc case), 16.8% (the German mark case), 40.6% (the Italian lira case), 15.5% (the Dutch guilder case), 17.2% (the Swedish krona case), 23.6% (the Swiss franc case), 15.2% (the British pound case), and 5.3% (the US dollar case). The rejection rates are generally low, suggesting that the ADF test tends not to find parity reversion when real exchange rates are governed by mean-reverting fractional processes. In only three of the eight fractional cases can the rejection rate exceed 20%. The fractional processes of the French franc, Italian lira and Swiss franc yield among the highest rejection rates. Interestingly, these are also the only cases where the ADF test can find no unit root in the actual data.

Cheung and Lai (2000) examine dollar-based real exchange rates using fractional analysis, as conducted here, and present evidence of mean reversion for many series of real dollar rates. Here an interesting study by Papell (1998) should also be noted. In showing how the long swings in the dollar during the 1980s can thwart the ability of researchers to unveil PPP reversion in real dollar rates, Papell (1998) proposes modeling the long dollar swings as exogenous events using multiple trend shifts. A panel test that allows for trend shifts under a constant-mean restriction is devised to test for parity reversion. It is shown that the unit-root null can be rejected in favor of a PPP-restricted structural change alternative. Significant evidence of PPP reversion can thus be uncovered in panel data, in contrast to other recent panel studies that report somewhat mixed or weak evidence (Engel et al., 1997; O'Connell, 1998; Papell, 1997). In considering statistical models that incorporate long-swing dynamics, the study here shares a similar spirit with Papell (1998), albeit the two studies have different modeling strategies.

#### 4. Further results from impulse response analysis

Although the foregoing findings confirm the empirical relevance of long-run PPP, they reveal little information on the adjustment process — including the pattern and speed — with which deviations from PPP die out. To obtain the relevant information, explicit computation of impulse responses becomes useful. Indeed, the persistence of real exchange rate dynamics can be analyzed through the sequence of  $C_j$ , as given by Eq. (5). The  $C_j$  function, which gives the sequence of  $C_j$  values at different time horizons after a unit shock, can be computed based on Eq. (3) for yen-based real exchange rates.

Graphs of the first 108 dynamic responses, corresponding to a time span of 9 years for monthly data of real yen rates, are displayed in Fig. 2. In all cases, the adjustment process has zero long-run persistence, confirming the existence of parity reversion. The process of convergence, however, exhibits interesting nonlinearity in the direction of adjustment due to short-term overreacting responses to the initial shock. All the graphs share a similar feature:  $C_j$  is not a monotonic function of the adjustment horizon. The  $C_j$  function is characterized by initial shock amplification

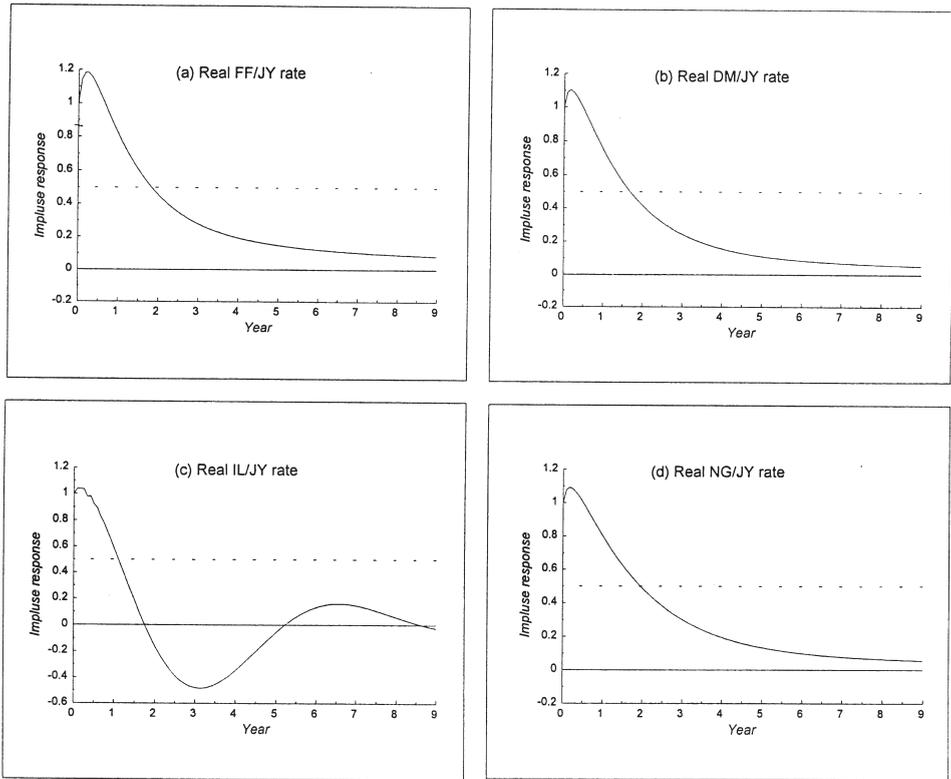


Fig. 2. Cumulative impulse responses estimated for yen-based real exchange rates.

before dissipation. Since the shock impact magnifies rather than diminishes in the initial phase, the maximum response cannot be felt until a few periods after the shock. Such sustained amplified responses can delay and prolong the process of convergence to parity. Moreover, among all the various real exchange rates, the IL/JY case depicts extra nonlinearity in a rather unique manner. In the IL/JY case, the short-term shock amplification is succeeded by significant undershooting before the shock impact eventually dampens out.

A measure of persistence typically applied in the PPP literature is the half-life, which indicates how long it takes for the impact of a unit shock on the real exchange rate to dissipate by half. The half-life can be computed from the  $C_j$  function as  $t=h$  at where  $C_h=0.5$ . The half-life estimates for real yen rates are reported in Table 5; they range from 1.1 to 2.5 years, except for the USD/JY case, which gives an estimate of about 2.9 years. The half-life estimates here are in most cases shorter than those reported for real dollar rates in prior studies — the half-life estimates have often been found to fall between 3 and 5 years (Froot and Rogoff, 1995; Rogoff, 1996). Table 5 also gives the half-life estimates of the speed at which PPP deviations dwindle after the full impact of short-term overreaction has happened, i.e., after  $C_j$

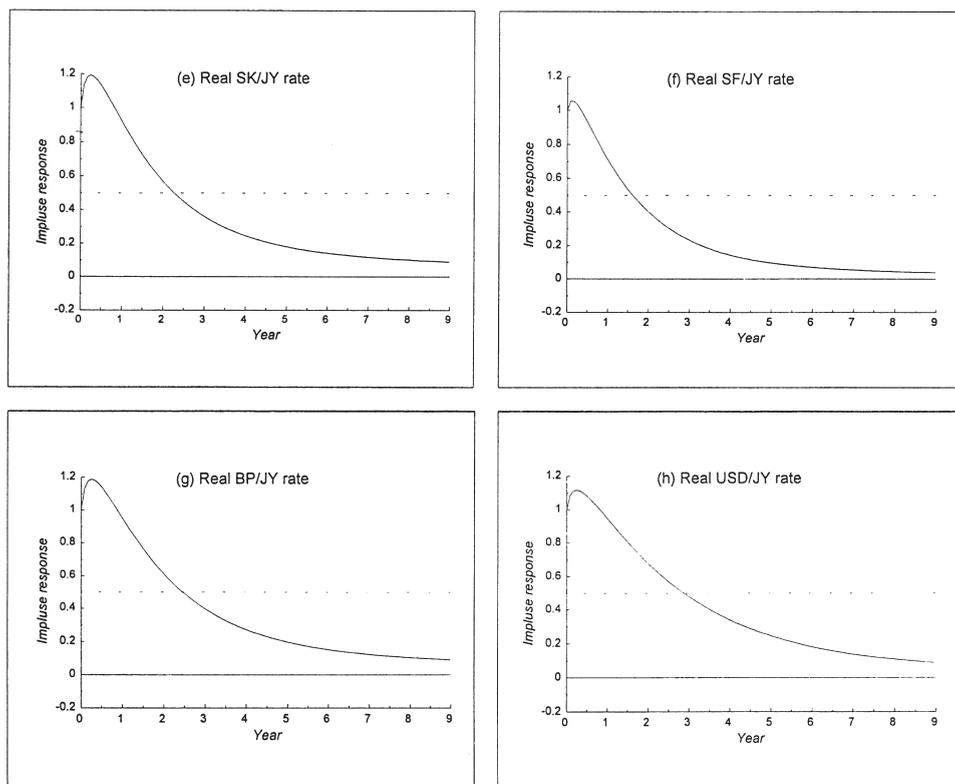


Fig. 2. (continued)

Table 5  
Half-life estimates for the speed of parity reversion

Exchange rate	Half-life estimates (years)	
	Initial overreaction included	Initial overreaction excluded
FF/JY	1.83	1.46
DM/JY	1.69	1.48
IL/JY	1.11	1.08
NG/JY	1.96	1.72
SK/JY	2.27	1.68
SF/JY	1.63	1.56
BP/JY	2.46	1.85
USD/JY	2.89	2.35

has reached a maximum value subsequent to the shock. The results indicate that after the impact of amplified responses has peaked, parity reversion in real yen rates takes place at a relatively fast speed. The implied non-constant speed of adjustment reflects the non-monotonicity in the short-term dynamic response.

Taylor et al. (1999) also identify variable speeds of parity reversion for a number of monthly real dollar rates under an interesting nonlinear framework. Using smooth transition autoregressive (STAR) models to analyze nonlinear adjustment, these researchers show that the adjustment speed increases with the size of the deviation from parity. For small shocks of 10% or less, STAR estimates of half-lives are found to be mostly between 2.5 to 3.5 years. For large shocks of 20% or more, on the other hand, STAR estimates of half-lives are reported to be about 1–2 years, much shorter than usual half-life estimates.

It should be noted that the short-term overreaction in shock response observed in this study cannot be fully explained by standard Dornbusch-type rational expectation models of overshooting. Indeed, in the Dornbusch (1976) model, short-term exchange rate overshooting occurs at the time of the shock only, and the real exchange rate reverts to its long-run equilibrium level monotonically. This implies that the impulse response function does not show a hump, as in Fig. 2, but peaks at the time of the shock ( $t=0$ ) and declines monotonically to zero. Unlike such overshooting dynamics, the amplified shock response here continues to enlarge PPP deviations even after the initial shock. The amplified response can contribute to the short-term volatility and add to the persistence of the real exchange rate.

The tendency for the shock impact to extend and amplify appears at variance with uncovered interest-rate parity (UIP), as pointed out by Eichenbaum and Evans (1995). A contractionary monetary shock in the US, for example, raises US interest rates relative to foreign interest rates. When the shock impact magnifies beyond the initial shock, the relative increase in the US interest rate will not be offset by an expected dollar depreciation, leading to persistent expected excess returns and systematic UIP deviations. Moreover, with the persistent appreciation of the dollar being associated with the higher interest-rate differential, the contractionary monetary shock will induce a negative forward premium bias, producing a negative correlation between the change in the exchange rate and the forward premium.

The chartist-fundamentalist model discussed by Frankel and Froot (1990, 1993) may provide a possible explanation for persistent UIP deviations. Exchange rate forecasts over short horizons may be dominated by chartists (trend-following traders with bandwagon expectations), albeit exchange rate forecasts over long horizons are governed by fundamentalists (fundamentals-based traders with regressive expectations). Following a contractionary monetary shock, for example, the domestic interest rate rises and the exchange rate (the price of foreign currency in terms of the domestic currency) falls. The decline in the exchange rate will continue for a while after the initial shock because there are chartists jumping on the bandwagon, selling the foreign currency and causing further depreciation. To the extent that short-term bandwagon effects exist, the impact of a market shock on exchange rates will tend to amplify before dissipating. The chartist–fundamentalist model is thus compatible with sustained deviations from UIP.

## 5. Conclusion

The extraordinary difficulty in uncovering evidence of parity reversion in real yen rates has often been ascribed to a missing trend variable, even though the inclusion of a time trend appears at odds with the usual notion of long-run PPP. This study explores an alternative explanation and shows that the puzzling behavior of real yen rates may stem from long-memory dynamics, which confound unit-root tests and undermine their ability to identify mean reversion. To the extent that long-memory dynamics are characterized by nonperiodic long cycles, the findings of such dynamics may reflect the long swings in yen exchange rates over the current float. It is shown that when the long-memory dynamics are properly accounted for using fractional time series models, strong evidence of parity reversion can be unveiled in real yen rates. Further analysis also reveals that real yen rates display non-monotonic mean reversion, with the rates moving further away from parity before reverting. In general, this study illustrates that a better understanding of the dynamic process under which the real exchange rate adjusts to a shock can be important for explaining the puzzling behavior of the real exchange rate.

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## Appendix A. Various trend-break tests for a unit root

There are several different AO models, capturing different types of trend breaks caused by either a change in the mean or the slope or both of the trend function. These models are represented by

$$y_t = c_0 + c_1 t + \theta d_t(n) + (1 - \rho L)^{-1} C(L) w_t, \tag{A1}$$

$$y_t = c_0 + c_1 t + \eta d_t^*(n) + (1 - \rho L)^{-1} C(L) w_t, \tag{A2}$$

$$y_t = c_0 + c_1 t + \theta d_t(n) + \eta d_t^*(n) + (1 - \rho L)^{-1} C(L) w_t, \tag{A3}$$

where  $d_t(n) = I(t > n)$ ;  $d_t(n) = (t - n)I(t > n)$ , with  $I(\cdot)$  being the indicator function; and  $(1 - \rho L)^{-1} C(L) w_t$  is a general innovation process. When  $\rho = 1$ ,  $y_t$  has a unit root. Specification (A1) permits a level shift to occur at time  $t = n$ , which implies a jump in the trend line induced by a mean shift. In contrast, a trend shift (i.e., a change in the slope of the trend function) is allowed for at  $t = n$  under specification (A2). Specification (A3) admits a shift in both mean and trend.

To test for a unit root under the AO models, detrended series are first obtained as follows:

$$y_t = \mu_0 + \mu_1 t + \phi d_t(n) + \tilde{y}_t^1, \tag{A4}$$

$$y_t = \mu_0 + \mu_1 t + \gamma d_t^*(n) + \tilde{y}_t^2, \tag{A5}$$

$$y_t = \mu_0 + \mu_1 t + \phi d_t(n) + \gamma d_t^*(n) + \tilde{y}_t^3, \tag{A6}$$

where the residual series,  $\tilde{y}_t^1$ ,  $\tilde{y}_t^2$  and  $\tilde{y}_t^3$  gives the detrended series of  $y_t$ . Next, tests for  $\beta_0=0$  under the null hypothesis of a unit root are performed using the following regressions:

$$(1-L)\tilde{y}_t^1 = \sum_{j=0}^p \omega_j D_{t-j}(n) + \beta_0 \tilde{y}_{t-1}^1 + \sum_{j=1}^p \beta_j (1-L)\tilde{y}_{t-j}^1 + e_t, \tag{A7}$$

$$(1-L)\tilde{y}_t^2 = \beta_0 \tilde{y}_{t-1}^2 + \sum_{j=1}^p \beta_j (1-L)\tilde{y}_{t-j}^2 + e_t, \tag{A8}$$

$$(1-L)\tilde{y}_t^3 = \sum_{j=0}^p \omega_j D_{t-j}(n) + \beta_0 \tilde{y}_{t-1}^3 + \sum_{j=1}^p \beta_j (1-L)\tilde{y}_{t-j}^3 + e_t, \tag{A9}$$

where  $e_t$  is the error term. The dummy variables,  $D_{t-j}(n) = I(t=n+j+1)$  for  $j=0, \dots, p$ , are included in cases involving a mean shift to ensure the test robustness with respect to the error correlation structure (Vogelsang and Perron, 1998). The break point,  $n$ , is assumed to be unknown and needs to be estimated from the data. Specifically, by varying  $n$  for each regression over the sample period,  $n$  will be chosen to maximize over a sequence of  $F$ -statistics testing the significance of the break parameters:  $\phi=0$  in regression (A4);  $\gamma=0$  in regression (A5); and  $\phi=0=\gamma$  in regression (A6). This method performs well in identifying the true break point. With  $T$  being the sample size, the  $t$ -statistics for testing  $\beta_0=0$  computed at where  $F(\tilde{n}, T) = \max_{r \leq n \leq T-r} F(n, T)$  are denoted by  $\tau_{DF}(AO, \tilde{n}, i)$ ,  $i=a, b$ , and  $c$ , for regressions (A7) to (A9). The trimming parameter,  $r$ , is set equal to the integer part of  $0.15T$ , following Banerjee et al. (1992).

In contrast to the AO approach, the IO approach entertains situations in which the break can occur not abruptly but slowly over time. In allowing for gradual structural changes, this approach provides more flexibility in modeling trend breaks than the AO approach. Various possible IO model specifications are:

$$y_t = c_0 + c_1 t + \psi(L)(\theta d_t(n) + v_t), \tag{A10}$$

$$y_t = c_0 + c_1 t + \psi(L)(\eta d_t^*(n) + v_t), \tag{A11}$$

$$y_t = c_0 + c_1 t + \psi(L)(\theta d_t(n) + \eta d_t^*(n) + v_t), \tag{A12}$$

where  $\psi(L)$  is a lag polynomial. Notice that through the function  $\psi(L)$ , a break can operate and impact the process gradually over a period of time. Parallel to the AO models, (A10) represents the mean-shift case, whereas (A11) is the trend-shift case. Specification (A12) combines the two cases together.

Corresponding to (A10), (A11), and (A12), we have the following regressions for unit-root tests:

$$(1-L)y_t = \mu_0 + \mu_1 t + \omega D_t(n) + \phi d_t(n) + \beta_0 y_{t-1} + \sum_{j=1}^p \beta_j (1-L)y_{t-j} + u_t, \tag{A13}$$

$$(1-L)y_t = \mu_0 + \mu_1 t + \gamma d_t^*(n) + \beta_0 y_{t-1} + \sum_{j=1}^p \beta_j (1-L)y_{t-j} + u_t, \quad (\text{A14})$$

$$(1-L)y_t = \mu_0 + \mu_1 t + \omega D_t(n) + \phi d_t(n) + \gamma d_t^*(n) + \beta_0 y_{t-1} + \sum_{j=1}^p \beta_j (1-L)y_{t-j} + u_t, \quad (\text{A15})$$

where  $u_t$  is the error term (see Vogelsang and Perron, 1998). Regression (A14) has been used by Banerjee et al. (1992) Zivot and Andrews (1992). These two studies also consider regressions similar to (A13) or (A15), but without the one-time dummy variable  $D_t(n)$ . As for the AO models, the break point,  $n$ , will be selected using the maxima of the  $F$ -statistics testing for  $\phi=0$  in regression (A13),  $\gamma=0$  in regression (A14), and  $\phi=0=\gamma$  in regression (A15). In each regression, the  $t$ -statistic for testing  $\beta_0=0$  is computed at the chosen break point,  $\tilde{n}$ , and the resulting statistic is denoted by  $\tau_{DF}(\text{IO}, \tilde{n}, j)$  for  $j=a, b$  or  $c$ .

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