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# Dissecting the PPP puzzle: the unconventional roles of nominal exchange rate and price adjustments

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## Abstract

The conventional view, as expounded by sticky-price models, is that price adjustment determines the PPP reversion rate. This study examines the mechanism by which PPP deviations are corrected. Nominal exchange rate adjustment, not price adjustment, is shown to be the key engine governing the speed of PPP convergence. Moreover, nominal exchange rates are found to converge much more slowly than prices. With the reversion being driven primarily by nominal exchange rates, real exchange rates also revert at a slower rate than prices, as identified by the PPP puzzle [J. Econ. Lit. 34 (1996) 647].

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## 1. Introduction

The theory of purchasing power parity (PPP) is notably silent on issues regarding the adjustment mechanism of its convergence process. The large, volatile PPP deviations observed after the advent of the modern float have bred the development of sticky-price models, which stress the role of slowly adjusting prices in determining the reversion rate. In his survey of the PPP literature, Rogoff (1996) points out that the observed persistence

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of real exchange rates is far too high to be explained by existing models of PPP deviations. Although growing evidence in support of PPP reversion has been documented (Frankel and Rose, 1996; Oh, 1996; Wu, 1996; Papell, 1997; Cheung and Lai, 1998, 2000a; Taylor and Sarno, 1998; Wu and Wu, 2001; Engel, 2000, gives an exception), consensus estimates of the reversion speed are remarkably slow with half-lives ranging from 3 to 5 years.<sup>1</sup> These half-lives seem too long to be explained by sticky-price models, which suggest faster reversion with half-lives of 1 to 2 years.

A recent study by Engel and Morley (2001) sheds new light on the issue in PPP convergence. These researchers observe that the root of the PPP puzzle may lie in the possibly different speeds of convergence for nominal exchange rates and prices. In contrast to standard rational-expectations sticky-price models, which impose the same reversion speed for nominal exchange rates and prices, they examine an empirical model that allows those variables to adjust at different speeds. Formulating the adjustment equations as a state-space model (Morley et al., 2003), Engel and Morley evaluate the speeds at which nominal exchange rates and prices converge to their respective equilibrium levels that are unobserved. Empirical results from state-space model estimation indicate that while prices converge relatively fast, nominal exchange rates converge slowly. The differing-speed finding is intriguing. It suggests that the torpid rate of PPP reversion may come largely from slow nominal exchange rate adjustment rather than from slow price adjustment. If it is true, the finding challenges conventional beliefs in the price-stickiness explanation and raises new issues in theoretical modeling of PPP disequilibrium dynamics.<sup>2</sup>

This study presents additional evidence on the convergence speeds of nominal exchange rates and prices. Using vector error correction (VEC) analysis, we estimate the speeds at which the individual variables revert to their long-run values. The VEC analysis provides an alternative, easier way to measure those convergence speeds than the state-space analysis does. The latter entails elaborate estimation of unobservable components. While taking a different empirical approach, our results corroborate those of Engel and Morley (2001) that nominal exchange rates do converge at a much slower rate than prices. Half-lives of nominal exchange rates are estimated to be from 3 to 6 years, whereas half-lives of prices are found to be substantially shorter—mostly about 1 to 2 years. This study also shows that about 60–90% of PPP disequilibrium adjustment takes place through nominal exchange rate adjustment. Hence, it is mostly nominal exchange rate adjustment—not price adjustment—that drives real exchange rates toward parity. As such, the observed rate of PPP reversion reflects primarily the speed of nominal exchange rate convergence. Should nominal exchange rates converge much more slowly than prices, the PPP reversion speed can be slower than the price convergence speed, as described by the PPP puzzle.

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<sup>1</sup> Cheung and Lai (2000b) and Murray and Papell (2002) illustrate the existence of substantial sampling variability in measuring half-lives. The present study is not concerned with the issue in measurement uncertainty. Instead, it takes the findings of extremely slow convergence as empirical facts and investigates how the slow convergence in real exchange rates can occur.

<sup>2</sup> Engel and Morley (2001) envision a possible role of herding behavior, which might send nominal exchange rates temporarily off onto disequilibrium paths, thereby prolonging the convergence.

The paper is organized as follows. Section 2 outlines the VEC model and presents some preliminary results. Section 3 evaluates the relative proportion of PPP adjustment attributable to nominal exchange rate and price adjustments. Section 4 analyzes relative convergence speeds. Section 5 concludes.

## 2. Basic empirical framework and preliminary results

Half-life estimates reported in the PPP literature have typically been obtained from univariate time-series analysis of the real exchange rate (denoted by  $q$ ):

$$B(L)q_t = \varepsilon_t \quad t = 1, 2, \dots \tag{1}$$

where  $L$  is the lag operator such that  $Lq_t = q_{t-1}$ ,  $B(L) = 1 - b_1L - \dots - b_kL^k$ , and  $\varepsilon_t$  is the random error. The real exchange rate, which captures the deviation from PPP, is measured by

$$q_t = e_t - p_t \tag{2}$$

where all variables are expressed in logarithms;  $e$  is the nominal exchange rate (the domestic price of foreign currency), and  $p$  is the relative ratio between the domestic price level ( $p^d$ ) and the foreign price level ( $p^f$ ). In studying a trivariate system of  $e$ ,  $p^d$ , and  $p^f$ , Engel and Morley (2001) report that  $p^d$  and  $p^f$  share similar convergence speeds, and so the theoretical symmetric condition holds. With the symmetric condition imposed, we consider a bivariate model of  $e$  and  $p$  for simplicity.<sup>3</sup> According to the Granger Representation Theorem (Engle and Granger, 1987), a cointegrated time-series system has an equivalent VEC representation. Let  $X_t = [e_t p_t]'$  and  $\beta = [1 - 1]'$ . The long-run PPP restriction on  $X_t$  is that  $\beta' X_t = e_t - p_t$  is stationary. The VEC model is in general given by

$$\Delta X_t = \mu - \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + u_t \tag{3}$$

where  $\Delta = 1 - L$ ,  $\Pi$  can be written as  $\Pi = \alpha\beta'$ , and  $u_t = [u_{1t} u_{2t}]'$  is a vector of white-noise innovations with  $E(u_t u_t') = \Omega$ . Specifically, the VEC model with the PPP restriction has the following structure:

$$\Delta e_t = \mu_1 - \alpha_1 z_{t-1} + \sum_{i=2}^k \tau_{1i} \Delta e_{t-i+1} + \sum_{m=2}^k \delta_{1m} \Delta p_{t-m+1} + u_{1t} \tag{4a}$$

$$\Delta p_t = \mu_2 + \alpha_2 z_{t-1} + \sum_{i=2}^k \tau_{2i} \Delta e_{t-i+1} + \sum_{m=2}^k \delta_{2m} \Delta p_{t-m+1} + u_{2t} \tag{4b}$$

<sup>3</sup> When experimenting with the trivariate system, we also found that home and foreign prices have similar adjustment dynamics. Modeling these prices separately did not produce any new useful results.

where  $z_{t-1} = \beta' X_{t-1} = q_{t-1}$  represents the error correction term with coefficients,  $1 > \alpha_1 > 0$  and  $1 > \alpha_2 > 0$ .

We first verify the long-run PPP relation. Monthly data on consumer price indices and exchange rates for five countries—Britain, France, Germany, Italy, and Japan—vis-à-vis the US are examined. Taken from the International Monetary Fund's *IFS* data CD-ROM, the data cover the period from April 1973 through December 1998. Panel A of Table 1 contains the results of Johansen's (1991) tests for cointegration. The results in all but one case support that  $e$  and  $p$  are cointegrated. The cointegration vector  $\beta$  cannot be uniquely identified under the VEC setting, however. Edison et al. (1997) find PPP restriction tests on  $\beta$  to have very poor size properties. As recommended by Froot and Rogoff (1995), the most direct way to verify the long-run PPP specification is to perform unit-root tests on the real exchange rate  $q$ . If  $q$  is stationary,  $e$  and  $p$  are cointegrated and have a VEC representation with  $\beta = [1 - 1]'$ . Elliott et al. (1996) devised the DF-GLS test, which is more efficient than usual unit-root tests. As reported in panel B, the unit-root null can be rejected in four of the five cases. In only one case (the case of Japan) do we fail to find stationarity. Because univariate unit-root tests may lack power, we also apply Taylor and Sarno's (1998) multivariate unit-root test, labeled as the JLR test, in which the null hypothesis is that at least one of the series in the panel is nonstationary. As shown in panel C, the null can be rejected at the 10% level, supporting that all the real exchange rate series under study are stationary. Accordingly, our analysis will proceed with the empirical model that the long-run PPP condition holds.

Table 1  
Results from cointegration and unit-root tests for stationarity

RHS variable	Britain/US		France/US		Germany/US		Italy/US		Japan/US		
	Lag	Statistic	Lag	Statistic	Lag	Statistic	Lag	Statistic	Lag	Statistic	
A. Cointegration test on $e_t$ and $p_t$ (Johansen, 1991)											
$-2 \ln Q_r$	2	26.736**	4	39.052**	2	19.959**	2	31.548**	6	12.250	
$-2 \ln Q_{rr+1}$	2	21.388**	4	34.716**	2	15.966**	2	26.621**	6	12.406	
B. Univariate unit-root test on $q_t$ (Elliott et al., 1996)											
DF-GLS	4	-1.750*	4	-2.280*	2	-2.039**	2	-2.207**	4	-0.687	
C. Multivariate unit-root test on $q_t$ (Taylor and Sarno, 1998)											
All five series in a panel											
JLR	3	3.001*									

The  $-2 \ln Q_r$  test gives Johansen's (1991) trace statistic, and the  $-2 \ln Q_{rr+1}$  test gives Johansen's (1991) maximal-eigenvalue statistic. The null hypothesis for these two tests here is that the data processes under consideration are not cointegrated. Critical values for both trace and maximum-eigenvalue statistics are given by Johansen and Juselius (1990) and Cheung and Lai (1993). The null hypothesis for the DF-GLS test is that the data process under examination contains a unit root. Critical values for the DF-GLS test are given by Elliott et al. (1996). The Taylor–Sarno (1998) JLR test statistic follows a known limiting  $\chi^2(1)$  distribution under the null hypothesis that at least one of the series in the panel contains a unit root. Rejecting this null will indicate that all the series under examination are stationary. Statistical significance is indicated by a single asterisk (\*) for the 10% level and a double asterisk (\*\*) for the 5% level.

Before examining the decomposed dynamics in terms of nominal exchange rate and price adjustments, it is useful to measure the half-life of PPP deviations under the VEC model. Following Pesaran and Shin (1996), the impulse response function of  $q_t$  (denoted

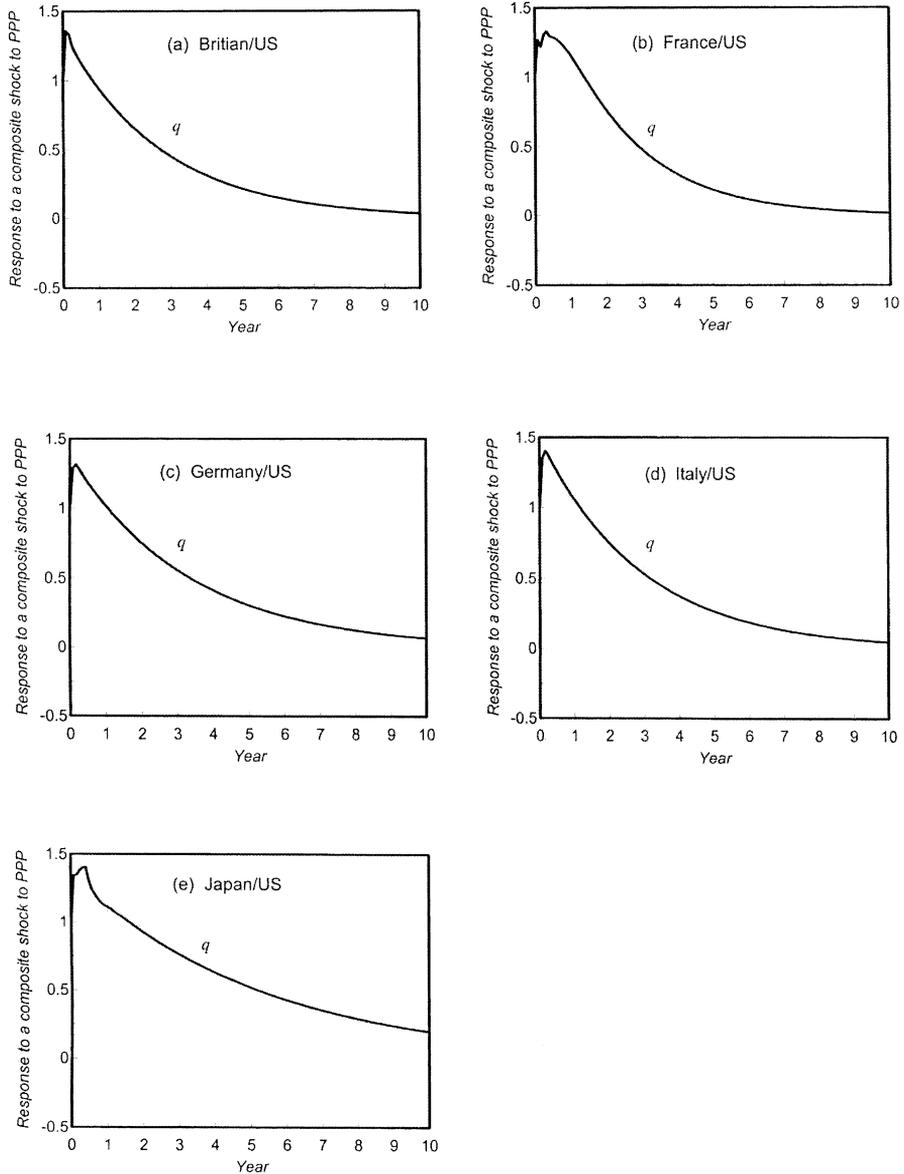


Fig. 1. Dynamic responses of real exchange rates to a composite innovation to PPP.

by  $\psi_{qc}$ ) with respect to a unit composite innovation  $\beta' u_t$  can be obtained from the VEC model as follows:

$$\psi_{qc}(t) = \{(\beta' C_t \Omega C_t' \beta)(\beta' \Omega \beta)^{-1}\}^{1/2} \quad (5)$$

where  $C_t$  is defined by a recursive equation:

$$C_t = A_1 C_{t-1} + A_2 C_{t-2} + \dots + A_k C_{t-k}, \quad t = 1, 2, \dots \quad (6)$$

with  $C_0 = I$  and  $C_t = 0$  for  $t < 0$ . The matrices  $\{C_t, t = 1, 2, \dots\}$  constitute the coefficient matrices of the moving-average representation of  $X_t$ . The VEC model specification is selected using the usual Akaike information criterion.<sup>4</sup> Based on  $\psi_{qc}$ , we compute the first 120 impulse responses, which correspond to a time span of 10 years for monthly data (see Fig. 1). The half-life of PPP deviations is estimated to be 2.81 years for the British pound, 3.02 years for the French franc, 3.43 years for the German mark, 3.26 years for the Italian lira, and 5.19 years for the Japanese yen. These half-life estimates are in line with the typical estimates of 3 to 5 years reported in the PPP literature.

### 3. Relative contributions of nominal exchange rate and price adjustments

We next decompose the real exchange rate dynamics and analyze the paths of nominal exchange rate and price adjustments separately. The generalized impulse response approach recommended by Pesaran and Shin (1998) is applied. Unlike traditional impulse response analysis (e.g. Lütkepohl and Reimers, 1992), which considers orthogonalized shocks based on the Cholesky decomposition, the new approach desirably yields unique impulse response functions (IRFs) that are invariant to the ordering of variables. The generalized IRF for  $X_t = [e_t p_t]'$  with respect to a unit innovation to the  $j$ th variable ( $j = 1$  for a nominal exchange rate innovation and 2 for a price innovation) is given by<sup>5</sup>

$$\psi_{X_j}(t) = C_t \Omega \gamma_j / \sigma_{jj}, \quad t = 0, 1, 2, \dots \quad (7)$$

where  $C_t$  is computed from (6) recursively,  $\gamma_j$  is a selection vector with unity as its  $j$ th element and zeros elsewhere, and  $\sigma_{jj}$  is the  $j$ th diagonal element of  $\Omega$ .  $\psi_{X_j}(t)$  gives the

<sup>4</sup> In each case we also experimented with VEC specifications of different lag orders,  $k=2, 3, 4, 5$ , and 6. The various lag specifications produced qualitatively similar results. The results reported later in this paper were found to be robust with respect to lag specifications.

<sup>5</sup> The central issue here concerns the different adjustment behavior of nominal exchange rates and prices, and we do not attempt to determine the structural sources of their innovations. Innovations to real exchange rates—real or monetary in nature—can operate through either nominal exchange rates or prices or both. Monetary changes, for instance, can affect both nominal exchange rates and prices. Hence, part of these innovations can come from similar sources, even though they work through different channels.

Table 2  
Relative contributions of nominal exchange rate and price adjustments to PPP reversion

	Under an exchange rate innovation				Under a price innovation			
	$t$	$g_{e1}$	$g_{p1}$	S.E. <sub>1</sub>	$t$	$g_{e2}$	$g_{p2}$	S.E. <sub>2</sub>
Britain/US	12	0.76	0.24	0.13	12	0.76	0.24	0.14
	24	0.76	0.24	0.13	24	0.76	0.24	0.13
	36	0.76	0.24	0.13	36	0.76	0.24	0.13
	60	0.76	0.24	0.13	60	0.76	0.24	0.13
	120	0.76	0.24	0.13	120	0.76	0.24	0.13
France/US	12	0.87	0.13	0.08	12	0.99	0.01	46.67
	24	0.91	0.09	0.05	24	0.90	0.10	0.11
	36	0.92	0.08	0.05	36	0.92	0.08	0.05
	60	0.92	0.08	0.05	60	0.92	0.08	0.05
	120	0.92	0.08	0.05	120	0.92	0.08	0.05
Germany/US	12	0.85	0.15	0.11	12	0.87	0.13	1.41
	24	0.85	0.15	0.11	24	0.85	0.15	0.11
	36	0.85	0.15	0.11	36	0.85	0.15	0.11
	60	0.85	0.15	0.11	60	0.85	0.15	0.11
	120	0.85	0.15	0.11	120	0.85	0.15	0.11
Italy/US	12	0.60	0.40	0.03	12	0.58	0.42	6.44
	24	0.59	0.41	0.03	24	0.59	0.41	0.17
	36	0.59	0.41	0.03	36	0.59	0.41	0.03
	60	0.59	0.41	0.03	60	0.59	0.41	0.03
	120	0.59	0.41	0.03	120	0.59	0.41	0.03
Japan/US	12	0.51	0.49	24.45	12	0.79	0.21	64.79
	24	0.68	0.32	0.04	24	0.70	0.30	0.31
	36	0.68	0.32	0.04	36	0.69	0.31	0.09
	60	0.68	0.32	0.04	60	0.68	0.32	0.06
	120	0.68	0.32	0.04	120	0.68	0.32	0.04

The time horizon  $t$  is measured in months. The columns  $g_{e1}$  and  $g_{e2}$  indicate the proportion of real exchange rate adjustment explained by nominal exchange rate adjustment. The columns  $g_{p1}$  and  $g_{p2}$  indicate the proportion explained by price adjustment. The column S.E.<sub>1</sub> provides the standard errors for both  $g_{e1}$  and  $g_{p1}$  estimates. The column S.E.<sub>2</sub> gives the standard errors for both  $g_{e2}$  and  $g_{p2}$  estimates. These standard errors are computed in simulation using the technique of resampling with replacement (based on 10,000 iterations), with the distributions of the innovation terms in the VEC model being approximated by the empirical distributions of the estimated residuals.

separate IRFs for nominal exchange rate and price adjustments (denoted by  $\psi_{ej}(t)$  and  $\psi_{pj}(t)$ , respectively). The generalized IRF for real exchange rate adjustment in response to a unit innovation to the  $j$ th variable is given by

$$\psi_{qj}(t) = \beta' C_t \Omega \gamma_j / \sigma_{jj}, \quad t = 0, 1, 2, \dots \quad (8)$$

A shock to PPP can come about as an exchange rate innovation or a price innovation. An increase in  $q$ , for example, can be induced by either a negative innovation to  $p$  or a

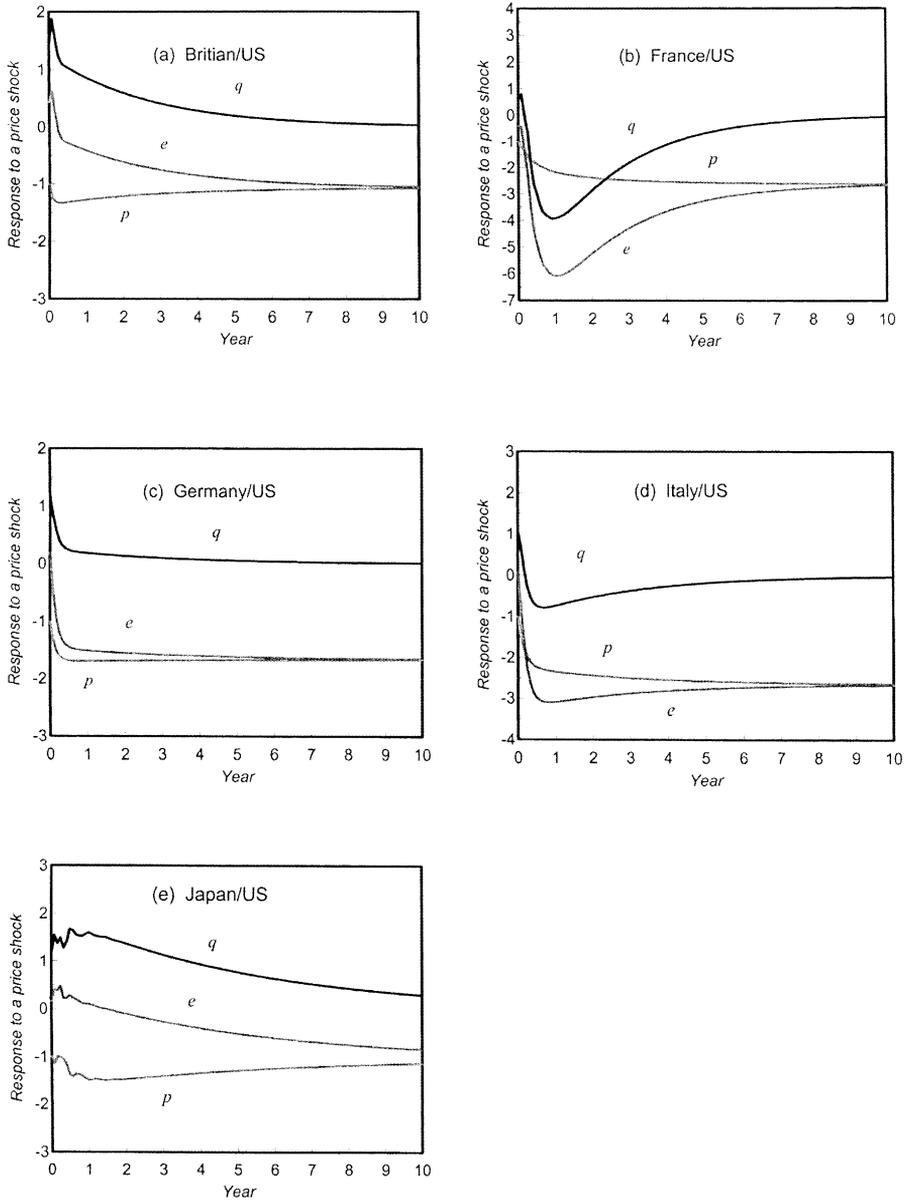


Fig. 2. Dynamic responses of real and nominal exchange rates and of prices to a price innovation.

positive innovation to  $e$ . In fact, the IRFs of  $q$ ,  $e$ , and  $p$  are linked to one another as follows:

$$\psi_{qj}(t) = \psi_{ej}(t) - \psi_{pj}(t), \quad j = 1, 2. \tag{9}$$

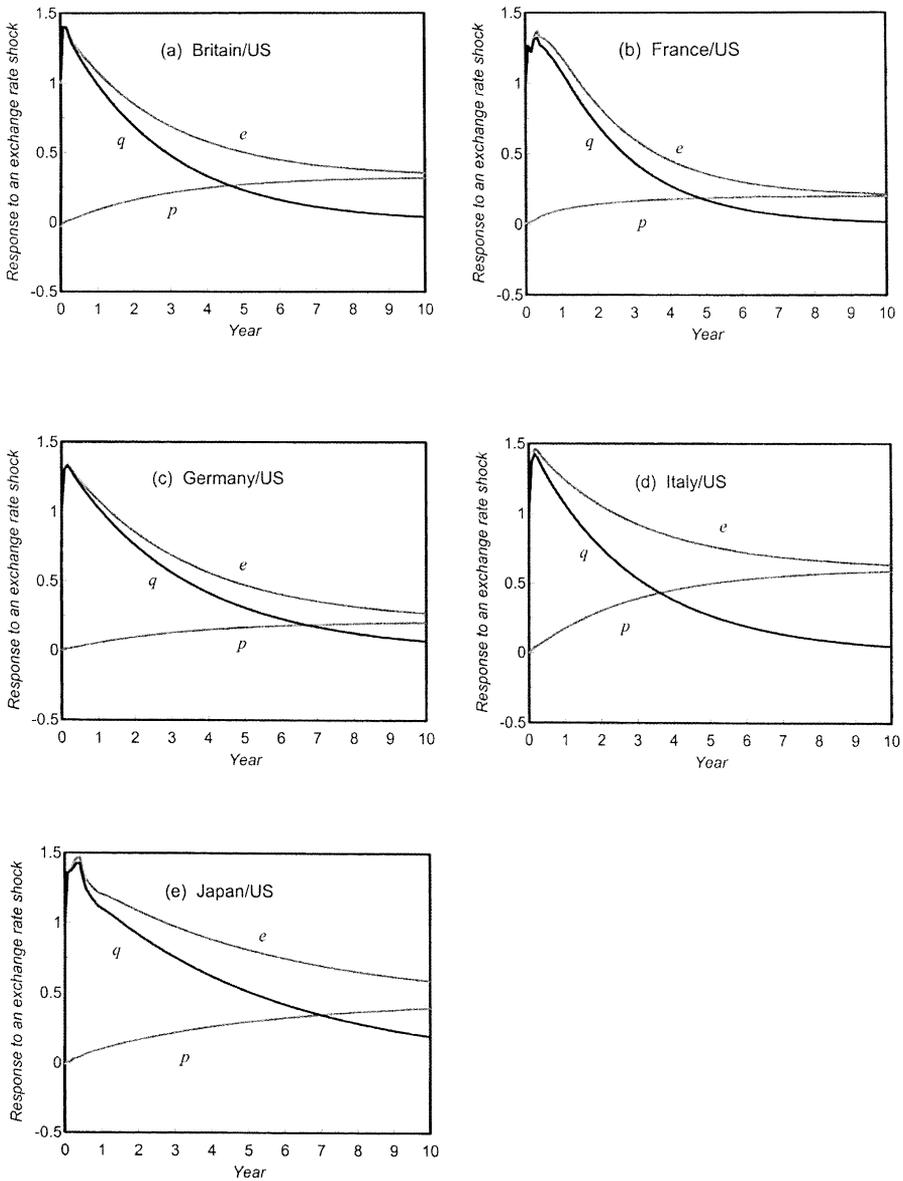


Fig. 3. Dynamic responses of real and nominal exchange rates and of prices to an exchange rate innovation.

To quantify the contributions of nominal exchange rate and price adjustments,  $\Delta\psi_{qj}(t)$  is broken into two components:  $\Delta\psi_{ej}(t)$  and  $\Delta\psi_{pj}(t)$  for  $j=1, 2$ . Letting  $g_{ej} = \Delta\psi_{ej}(t)/\Delta\psi_{qj}(t)$  and  $g_{pj} = -\Delta\psi_{pj}(t)/\Delta\psi_{qj}(t)$ , we have  $g_{ej} + g_{pj} = 1$ , where  $g_{ej}$  gives the proportion of real exchange rate adjustment explained by nominal exchange rate adjustment, and  $g_{pj}$  measures the proportion explained by price adjustment.

Table 2 reports the decomposition estimates, along with their standard errors, for different time horizons subsequent to a shock. Given that  $g_{ej} + g_{pj} = 1$ , the  $g_{e1}$  and  $g_{p1}$  estimates are of equal standard errors and so are the  $g_{e2}$  and  $g_{p2}$  estimates. We observe that standard errors of estimates for short horizons of 12 months or less can sometimes be very large, suggesting a high level of uncertainty in estimating short-run adjustment dynamics. At longer horizons, nonetheless, the differences between  $g_{ej}$  and  $g_{pj}$  estimates are statistically significant. Specifically, the  $g_{ej}$  estimates are generally much larger than the  $g_{pj}$  estimates, supporting that PPP deviations are corrected mainly through nominal exchange rate adjustment, albeit price adjustment also partly contributes to restoring parity. In relative magnitude, approximately 60–90% of the PPP reversion dynamics occur through nominal exchange rate adjustment. The results are independent of whether the shock operates as a nominal exchange rate or price innovation.<sup>6</sup>

The IRFs with respect to a price innovation are exhibited in Fig. 2. In every case, the shape of the IRF for  $q$  largely reflects that of the IRF for  $e$ , confirming that the nominal exchange rate is the prime engine for PPP reversion. Furthermore, if price innovations were dominating, we would expect the IRF for  $q$  under price innovations to be very similar to the IRF for  $q$  under composite innovations (i.e. the IRF for  $q$  under both price and exchange rate innovations together). Evidently, little similar dynamics can be observed between  $\psi_{q2}$  and  $\psi_{qc}$ . Indeed,  $\psi_{q2}$  differs dramatically from  $\psi_{qc}$  in most cases, indicating that price innovations are a relatively unimportant source of disturbances buffeting real exchange rates.

The IRFs of  $q$ ,  $e$ , and  $p$  in response to a nominal exchange rate innovation are displayed in Fig. 3. In all the cases the hump-shaped adjustment patterns are very much alike. Again, the shape of the IRF for  $q$  mostly reflects that of the IRF for  $e$ , supporting that the PPP reversion is governed primarily by nominal exchange rate adjustment. Moreover, the predominance of nominal exchange rate innovations can be verified by comparing  $\psi_{q1}$  (the IRF for  $q$  following an exchange rate innovation) with  $\psi_{qc}$  (the IRF for  $q$  following a composite innovation). Consistently,  $\psi_{q2}$  matches  $\psi_{qc}$  very closely, indicating that nominal exchange rate disturbances are the dominant source of real exchange rate fluctuations.<sup>7</sup> Further evidence in support of the relative importance of nominal exchange rate innovations comes from half-life calculation. Half-lives of real exchange rates under nominal exchange rate innovations are estimated to be 2.80 years for the British pound, 2.75 years for the French franc, 3.46 years for the German mark, 3.19 years for the Italian lira, and 5.08 years for Japanese yen. These estimates are almost the same as those half-life estimates computed earlier under composite innovations.

#### 4. Relative convergence speeds of nominal exchange rates and prices

With the nominal exchange rate being the main propeller of PPP reversion, the observed rate of PPP reversion should reflect much more the speed of nominal exchange rate

<sup>6</sup> In their study of many different real exchange rate appreciation episodes, Goldfajn and Valdes (1999) have found that most of the overvaluation is often corrected through nominal exchange rate adjustment rather than price adjustment.

<sup>7</sup> This is also supported by our error variance estimates that the standard deviation of nominal exchange rate innovations is about 4 to 8 times higher than that of price innovations.

adjustment than the speed of price adjustment. The convergence of the real exchange rate can therefore be sluggish if the nominal exchange rate converges slowly. Empirical speed estimates will show this is indeed the case.

Given the predominance of nominal exchange rate innovations in real exchange rate dynamics, we examine the IRFs in  $\psi_{q1}$ ,  $\psi_{e1}$ , and  $\psi_{p1}$  more closely. At the PPP equilibrium, we have

$$\psi_{q1}(t^*) = \psi_{e1}(t^*) - \psi_{p1}(t^*) = 0 \quad (10)$$

at time  $t = t^*$ . By examining the adjustment paths of individual variables (i.e.  $\psi_{q1}(t) \rightarrow 0$ ,  $\psi_{e1}(t) \rightarrow \psi_{e1}(t^*)$ , and  $\psi_{p1}(t) \rightarrow \psi_{p1}(t^*)$ ) subsequent to an innovation at time  $t=0$ , we can measure how fast these variables adjust and converge to their respective long-run equilibrium values. Analytically, the convergence takes place asymptotically (i.e. as  $t^* \rightarrow \infty$ ) and

$$\psi_{e1}(t_\tau) > \psi_{e1}(t^*) = \psi_{p1}(t^*) > \psi_{p1}(t_\tau) \text{ for } t_\tau < \infty. \quad (11)$$

In finite-sample estimation,  $\psi_{e1}(t^*)$  and  $\psi_{p1}(t^*)$  can be estimated based on a sufficiently large  $t_\tau$ . In our case  $\psi_{e1}(t^*)$  and  $\psi_{p1}(t^*)$  will be estimated as follows:

$$\psi_{e1}(t^*) = \psi_{p1}(t^*) = \{\psi_{e1}(t_\tau) + \psi_{p1}(t_\tau)\}/2 \quad (12)$$

where  $t_\tau = 240$  months and  $|\psi_{e1}(t) - \psi_{p1}(t)|$  is very close to zero as  $t \rightarrow t_\tau$ .

Half-life estimates of the convergence of  $p$ ,  $e$ , and  $q$  are presented in Table 3. These convergence speed estimates confirm that real exchange rates converge substantially slower than prices—the half-life of real exchange rate convergence ( $HL_q$ ) is about 1.5 to

Table 3  
Half-lives of convergence of nominal exchange rates, prices, and real exchange rates

	Britain/ US	France/ US	Germany/ US	Italy/ US	Japan/ US
Real exchange rate convergence:					
$HL_q$ [half-life in years]	2.80	2.75	3.38	3.19	5.08
Nominal exchange rate convergence:					
$HL_e$ [half-life in years]	3.21	3.03	3.62	4.40	6.27
Price convergence:					
$HL_p$ [half-life in years]	1.79	1.00	2.32	2.02	3.19
Difference in convergence speed:					
$HL_e - HL_p > 0$	1.42	2.03	1.30	2.38	3.08
(Standard error of $HL_e - HL_p$ )	(1.01)	(0.72)	(0.77)	(0.98)	(1.57)

Standard errors for the difference in half-life estimates,  $HL_e - HL_p$ , are computed in simulation using the technique of resampling with replacement (based on 10,000 iterations), with the distributions of the innovation terms in the VEC model being approximated by the empirical distributions of the estimated residuals.

2.5 times longer than the half-life of price convergence ( $HL_p$ ). Rogoff (1996, p. 654) discusses the PPP puzzle as follows:

The failure of short-run PPP can be attributed in part to stickiness in nominal prices; as financial and monetary shocks buffet the nominal exchange rate, the real exchange rate also changes in the short run. This is the essence of Dornbusch's (1976) overshooting model of nominal and real exchange rate volatility. If this were the entire story, however, one would expect substantial convergence to PPP over 1 to 2 years, as wages and prices adjust to shock.

Our speed estimates, indeed, show that prices converge at a reasonably fast rate with half-lives of about 2 years or less. The only exception is the case of Japan, for which the half-life of price convergence is roughly 3 years but for which the half-life of real exchange rate convergence takes even longer—in excess of 5 years. All the convergence speed estimates thus support that the slow convergence of real exchange rates does not stem from slowly converging prices. Instead, the slow PPP convergence comes from slowly converging nominal exchange rates. Half-lives of nominal exchange rate convergence ( $HL_e$ )—which range from 3 to 6 years—are consistently longer (about 1.5 to 3 times longer) than those of price convergence.<sup>8</sup> Standard errors of  $HL_e - HL_p$  estimates are computed using the simulation method, and the observed half-life differences are significantly greater than zero (i.e.  $HL_e - HL_p > 0$ ) at the 5% level in most cases and at the 10% level in all cases.

Our results here corroborate and reinforce those reported by Engel and Morley (2001), who also uncover surprisingly slow convergence for nominal exchange rates. Since the Engel–Morley study and our study have used different empirical approaches to estimate the disequilibrium adjustment dynamics, it is instructive to identify the key differences between the two approaches. For the discussion purpose, the basic structure of the Engel–Morley model (with the symmetric condition imposed) can be captured as follows:

$$e_t - E_{t-1}[\bar{e}_t] = \phi_1(e_{t-1} - \bar{e}_{t-1}) + \omega_{1t}, \quad 0 < \phi_1 < 1 \quad (13a)$$

$$p_t - E_{t-1}[\bar{p}_t] = \phi_2(p_{t-1} - \bar{p}_{t-1}) + \omega_{2t}, \quad 0 < \phi_2 < 1 \quad (13b)$$

where the overbars indicate the unobserved equilibrium values of the corresponding variables and  $\omega_{1t}$  and  $\omega_{2t}$  are random errors. The PPP condition,  $\bar{e}_t = \bar{p}_t$ , gives a cross-

<sup>8</sup> Since the convergence of  $q$  (i.e.  $q_t \rightarrow \bar{q}$ ) is governed by a linear combination of  $e$  and  $p$  adjustments (i.e.  $e_t \rightarrow \bar{e}$  and  $p_t \rightarrow \bar{p}$ ), where  $q_t - \bar{q} = (e_t - \bar{e}) + (\bar{p} - p_t)$ , the speed of real exchange rate convergence should fall between the speed of nominal exchange rate convergence and that of price convergence. The results in Table 3 bear out this expected pattern of differential speeds. Consistently, the  $HL_q$  estimates are shorter than the  $HL_e$  estimates but longer than the  $HL_p$  estimates.

equation restriction. In specifying the equilibrium processes, the first differences of  $\bar{e}_t$  and  $\bar{p}_t$  are considered to follow autoregressive processes. With some additional structural restrictions imposed, the entire system can be written as a state-space unobservable components model and estimated using the Kalman filter method.

The Engel–Morley state-space analysis and our VEC analysis have different notions of convergence. In the Engel–Morley model, nominal exchange rates and prices are considered as converging toward their moving equilibrium levels,  $\bar{e}_t$  and  $\bar{p}_t$ , respectively. While nominal exchange rates adjust to reduce the gap between  $e_t$  and  $\bar{e}_t$ , prices adjust to reduce the gap between  $p_t$  and  $\bar{p}_t$ . Accordingly, the disequilibrium is gauged by both  $e_t - \bar{e}_t$  and  $p_t - \bar{p}_t$  at time  $t$ . In our VEC model, on the other hand,  $e_t$  and  $p_t$  are both modeled as converging toward their long-run (stationary) equilibrium value,  $\bar{e} = \bar{p}$ . The disequilibrium is then represented by both  $e_t - \bar{e}$  and  $p_t - \bar{p}$  at time  $t$ . Empirical estimates of convergence speeds from the Engel–Morley study and our study should thus be interpreted with the different notions of convergence in mind.

The difference in convergence modeling has implications for model estimation. For the Engel–Morley model, the moving equilibrium values,  $\bar{e}_t$  and  $\bar{p}_t$ , are unobserved stochastic variables. As Charles Engel has observed, estimating the Engel–Morley model is not straightforward, and statistical results are conditional on the proper identification and estimation of the unobserved variables. The relative simplicity of the VEC model, by contrast, enables straightforward estimation of convergence speeds. The VEC model is derived from a general, direct decomposition of real exchange rate dynamics based on the Granger Representation Theorem for cointegrated time-series systems. It does not impose any specific structural restrictions other than the long-run PPP condition. The VEC model also requires no identification and estimation of any unobserved stochastic components. The total deviation from long-run PPP is measured by  $e_t - \bar{e} + p_t - \bar{p} = e_t - p_t$ , which can be directly observed.

## 5. Concluding remarks

This study has investigated the mechanism by which PPP deviations are corrected. Using an empirical approach different from Engel and Morley (2001), our VEC analysis produces further evidence for the difference in convergence speed between nominal exchange rates and prices. The VEC approach provides a convergence speed measure that requires easier estimation—and is probably less model-specific—compared to Engel and Morley’s state-space approach. Although there are significant differences between the two approaches, our results corroborate and reinforce Engel and Morley’s surprising finding: it is nominal exchange rates, not prices, that converge slowly toward PPP. The robustness of the finding calls into question the basic tenet of traditional models of PPP deviations, which emphasize price stickiness as the key determinant of the convergence speed. If the sluggish-price-adjustment explanation is empirically significant, we should not find prices to converge faster than both nominal and real exchange rates. Accordingly, the PPP puzzle should be rethought to recognize

the pivotal role nominal exchange rate adjustment plays in determining the PPP reversion rate.<sup>9</sup>

As a caveat, this study does not explore possible nonlinearities in PPP convergence—an interesting area for further research. While inviting alternative explanations for the PPP puzzle, Rogoff (1996) posits that trade costs (e.g. transport costs and tariffs) may be a contributing factor. Because of goods-market frictions, there is a band within which nominal exchange rates can move without eliciting a quick response in relative prices. With torpid price adjustment, real exchange rates converge very slowly inside the band. The trade-costs view has gained popularity and prompted a number of studies on nonlinear PPP reversion (e.g. Michael et al., 1997; Obstfeld and Taylor, 1997; O'Connell, 1998; Taylor et al., 2001; O'Connell and Wei, 2002). The transaction-costs-cum-nonlinearity explanation is instructive and useful in highlighting the significance of goods-market impediments to price adjustment.

Engel and Morley (2001) have succinctly stated that the real puzzle is 'why nominal exchange rates converge so slowly'. As such, it is not clear how transaction costs, which should be relatively insignificant for foreign exchange markets, can account for the slow convergence of nominal exchange rates. This poses a new challenge to theoretical models to explain the surprising behavior of nominal exchange rates. There is actually more to the puzzle. Why are the convergence rates of prices and nominal exchange rates different? Can heterogeneous convergence speeds be consistent in general equilibrium? Conventional models of PPP disequilibrium adjustment are based on saddle path analysis under rational expectations (à la Dornbusch's (1976) sticky-price models). Saddle path analysis typically prescribes that the endogenous (state and costate) variables—which are prices and nominal exchange rates in our case—will both converge to the steady state at the same rate. The empirical evidence suggests this is not the case, however. The differing speeds of convergence thus constitute a special puzzle that calls for new explanations.<sup>10</sup>

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<sup>9</sup> The results from this study—and also the Engel–Morley study—make an interesting contrast with those presented by Cecchetti et al. (2002). Cecchetti et al. find PPP convergence between cities within the US to be much slower than that observed between nations. This suggests the nominal exchange rate may facilitate adjustment toward PPP. Instead of showing a facilitating role, our results indicate that the nominal exchange rate prolongs PPP disequilibrium adjustment.

<sup>10</sup> The authors owe this insight to two anonymous referees.

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