Long-term Persistence in the Real Interest Rate: Some Evidence of a Fractional Unit Root

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This study examines the long-term persistence in *ex ante* real interest rates. According to the long-run Fisher effect, *ex ante* real rates—the difference between nominal rates and expected inflation—should be mean-reverting and have no unit root. Empirical evidence on mean reversion has been mixed and less than supportive, however. Prior analyses are restricted to integer orders of integration only. This study provides a re-appraisal of the evidence using fractional integration analysis. In addition, expected inflation is measured by inflation forecasts and not just by realized inflation rates. Empirical results strongly support that *ex ante* real interest rates exhibit mean reversion, but in a special manner not captured by the usual stationary processes. This finding is also corroborated by empirical results based upon *ex post* real rates.

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SUMMARY

This study investigates the question of whether the real interest rate displays mean reversion. According to the Fisher equation, the nominal interest rate and expected inflation move together one-for-one in the long run. For this to hold true, the real interest rate, which is the difference between the nominal rate and expected inflation, should be mean-reverting. Assuming that the inflation forecast error is stationary, prior studies typically examine the behaviour of *ex post* rather than *ex ante* real interest rates. Most of the evidence reported so far has suggested that the real interest rate is non-stationary and contains a unit root component (i.e. it is integrated of order one), a finding that is inconsistent with much theoretical work.

The present study is motivated by the observation that statistical analyses in prior studies in the literature are generally restricted to integer orders of integration only. Instead of imposing the standard assumption that the process for the real interest rate has exactly one or no unit root, the study allows for a fractional unit root to capture subtle mean-reverting dynamics. In addition, expected inflation is measured by inflation forecasts and not just by realized inflation rates. Considerable evidence is found to support that *ex ante* real interest rates computed using inflation forecasts contain not an exact unit root but a fractional one. The finding implies that real interest rates display mean-reverting dynamics, but in a special manner not captured by usual stationary processes. This finding is also corroborated by empirical results

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based upon *ex post* real rates computed using actual inflation.

1. INTRODUCTION

The *ex ante* real interest rate is a crucial variable determining valuations of financial assets and influencing macroeconomic dynamics. Although this variable can show significant fluctuations over time, its long-run behaviour is linked to a long-run relationship between nominal interest rates and expected inflation. According to the Fisher (1930) equation, nominal interest rates and expected inflation move together one-for-one in the long run. For the long-run Fisher relationship to hold, the *ex ante* real rate—the difference between the nominal rate and expected inflation—should display mean reversion. However, since the work of Rose (1988), who reported evidence of a unit root in real interest rates, the mean-reverting property of the *ex ante* real rate has been called into question. Rose (1988) also noted that the presence of a unit root in the real interest rate is inconsistent with Lucas-type consumption-based asset pricing models (Hansen and Singleton, 1982, 1983; Lucas, 1978), given that the growth rate of consumption has been found to contain no unit root. In earlier work, the presence of a unit root was indeed imposed without formal testing in time-series modelling of real interest rates (Antoncic, 1986; Fama and Gibbons, 1982; Garbade and Wachtel, 1978).

Recent empirical evidence has often indicated that nominal interest rates and realized inflation are driven by permanent shocks (Bonham, 1991; Evans and Lewis, 1995; King and Watson, 1992; MacDonald and Murphy, 1989; Mishkin, 1992, 1995). The findings of permanent disturbances underlying both variables confer special importance on examining mean reversion in real rates. Unless nominal rates and rationally expected inflation respond one-for-one to permanent shocks, *ex ante* real rates will be affected by the same permanent shocks as expected inflation, contradicting the long-run superneutrality of money (King and Watson, 1992; Evans and Lewis, 1995).

Two approaches are typically employed to study mean reversion in real interest rates. One examines cointegration between nominal interest rates and realized inflation under the assumption of a stationary forecast error. If the nominal rate and realized inflation are non-stationary I(1) processes (i.e. they are integrated of order one) but cointegrated of order CI[1, 1], then the *ex ante* real rate is an I(0) process and thus mean-reverting, provided that the cointegration coefficient is unity. The latter condition is required for a full long-run Fisher effect (Mishkin, 1992). The other approach involves testing directly for a unit root in real rates. This is equivalent to testing for cointegration between nominal rates and inflation, with a unity coefficient being imposed.

Empirical evidence on cointegration between nominal interest rates and realized inflation has been mixed so far. The cointegration relationship, even if found, has a slope coefficient considerably different from one. The results on cointegration thus provide not much support for mean reversion in real interest rates. Direct tests for a unit root in *ex post* real rates also yield less than supportive evidence for mean reversion. These results are mostly based on conventional tests such as Dickey–Fuller tests, which are known to have low power against persistent autoregressive alternatives or fractionally integrated alternatives (Diebold and Rudebusch, 1991a; Sowell, 1990). The latter includes a rich class of processes with slow mean reversion.

Evans and Lewis (1995) suggested that the use of data on realized inflation, common in previous studies, can produce substantial small-sample bias in estimates of the Fisher relationship due to the so-called 'peso problem'. For example, the market can rationally anticipate possible shifts in the inflation process, which may not materialize eventually. Accordingly, deviations between expected and realized inflation can be highly persistent, creating the appearance of permanent shocks to real interest rates. These authors examined inflation forecasts generated by a Markov switching model that allows for mean shifts in the inflation process. The generated inflation forecasts are shown to give long-run coefficient estimates much closer to one than actual inflation data, albeit the improved estimates are all still below 0.8. To be sure, biases in estimating long-run coefficients in finite samples may often exist. Banerjee *et al*. (1986) observed that cointegrating regressions in general can yield a sizeable small-sample bias, making coefficient estimates none too reliable. Mishkin (1995) illustrated the potential bias in the interest-rate–inflation regression.

The analysis by Evans and Lewis (1995) is instructive. It shows that the real interest rate appears mean-reverting after allowing for non-linearities in the underlying data process. The present study explores whether the real interest rate can be modelled as a fractionally integrated process. Since non-linearities may be picked up as fractional dynamics, the findings here can be interpreted as being complementary to those of Evans and Lewis (1995). In going beyond the usual statistical framework, fractional dynamics represent a plausible alternative to the hypothesis of structural shifts.

The mixed findings on the long-run behaviour of real interest rates reflect a basic problem in identifying mean reversion. A test of mean reversion entails proper modelling of the low-frequency dynamics, while allowing for possibly persistent dynamics in the short run. Consequently, empirical results can depend critically on the power of the statistical technique applied to separate the low-frequency from the high-frequency dynamics. Generic unit root tests presume the integration order to be an integer: I(1) or I(0). Admissible mean-reverting dynamics are thus restricted to I(0) processes solely. However, more general processes of fractional integration, I(d) with $d < 1$, are also mean-reverting, although their dynamics can be rather persistent (Diebold *et al.*, 1991; Cheung and Lai, 1992). By avoiding the stringent I(1)/I(0) distinction, fractional integration analysis permits a wider range of mean-reverting behaviour than unit root analysis.

This study analyses the long-run behaviour of the real interest rate using fractional integration analysis. Empirical evidence supports that the real rate displays mean reversion, but in a special manner not captured by I(0) processes. The non-I(0) dynamics may partly explain the mixed and less than supportive findings on mean reversion reported in the literature. Tests for fractional integration reveal that both the I(1) hypothesis and the I(0) hypothesis are consistently rejected by the data on real interest rates. The evidence of non-I(0) behaviour is also corroborated by results from a test for stationarity. The results in general suggest that the real interest rate follows an I(d) process with $0 < d < 1$. To the extent that the nominal rate and expected inflation are I(1) series, the finding of I(d) behaviour for the real rate further implies that the nominal rate and expected inflation are fractionally cointegrated (Cheung and Lai, 1993). The implied order of cointegration is CI[1, $1 - d$], in contrast to the usual CI[1, 1] system considered in cointegration analysis. Such a difference in the cointegration order has important implications for the rate of convergence for estimates of cointegrating coefficients, which can very much affect the potential accuracy in estimating the cointegrating coefficients in finite samples.

Inflation forecasts are employed to measure expected inflation in this study. This contrasts with most studies in the literature which use actual inflation rates to construct real interest rates. Nevertheless, the finding of fractional dynamics in real interest rates is shown to be robust with respect to whether inflation forecasts or actual inflation rates are examined.

The paper is organized as follows. Section 2 discusses the empirical specification of the long-run Fisher relationship. Section 3 describes the data under study on *ex ante* real interest rates. Section 4 contains results from conventional unit-root tests. Section 5 reports alternative results from fractional integration analysis. Section 6 provides further empirical evidence based on *ex post* real interest rates. Section 7 concludes.

2. THE LONG-RUN FISHER RELATIONSHIP

According to Fisher (1930), the one-period nominal interest rate at time t, denoted by i_t, can be broken into two components as follows:

$$i_t = r_t + \pi_t^e, \tag{1}$$

where r_t is the *ex ante* real interest rate and π_t^e is the expected inflation rate. If changes in π_t^e have no permanent effects on r_t, those changes in π_t^e should be reflected fully in subsequent movements of i_t over time, thereby resulting in a one-for-one relationship between i_t and π_t^e in the long run. This relationship has been called the long-run Fisher relationship. When i_t and π_t^e are non-stationary I(1) series, the long-run Fisher effect is testable in a cointegrating regression, given by

$$i_t = \alpha + \beta\pi_t^e + z_t, \tag{2}$$

where z_t is the error term. The long-run Fisher effect exists if $\beta = 1$ and i_t and π_t^e are cointegrated of order CI[1, 1], i.e. z_t is I(0). Since $r_t = i_t - \pi_t^e$ and can be written as

$$r_t = \alpha + (\beta - 1)\pi_t^e + z_t, \tag{3}$$

the conditions of $\beta = 1$ and CI[1, 1] are equivalent to r_t being an I(0) process, with no permanent component shared by π_t^e.

The requirement that r_t be I(0) appears arbitrary and is not necessary for the long-run Fisher relationship to hold. An equilibrium relationship can prevail in the long run as long as deviations from the equilibrium relationship follow a mean-reverting I(d) process with $d < 1$. Hence, a more general condition for the long-run Fisher effect is that r_t is I(d) with $d < 1$ or, equivalently, $\beta = 1$ and i_t and π_t^e are fractionally cointegrated of order CI[1, $1 - d$], i.e. z_t is I(d) with $d < 1$. For the latter, Cheung and Lai (1993) showed that the convergence rate in probability, at which consistent estimates of β can be obtained, is given by $O(T^{1-d})$, which is slower than the regular rate of $O(T)$ under cointegration of order CI[1, 1]. The closer is the value of d to unity, the greater will be the difference in the convergence rate. For $d = 0.8$, as shown in the data later, the convergence rate for estimating β for a CI[1, $1 - d$] system is given by $O(T^{0.2})$, a much slower rate than $O(T)$. This suggests that if i_t and π_t^e are in fact fractionally cointegrated with d less than but near unity, estimation of β will be highly imprecise in finite samples, making the β estimates unreliable and hard to interpret. In this study, the direct approach to test for I(d) behaviour in r_t is adopted.

In testing for the long-run Fisher effect, a common problem is that expected inflation is not directly observable. As a result, researchers have to rely on some proxies for π_t^e, say $\tilde{\pi}_t^e$, such that

$$\tilde{\pi}_t^e = \pi_t^e + u_t, \tag{4}$$

where the measurement error, u_t, is assumed to be stationary. Realized inflation is often used as a proxy for π_t^e; in this case, $\tilde{\pi}_t^e = \pi_t$ and u_t represents the forecast error. Underlying this practice is usually the assumption of rational expectations, under which u_t is an innovation orthogonal to the information set available when expectations are formed at time t. Such an assumption is too stringent and, indeed, not necessary for the validity of the long-run Fisher relationship. Instead, the minimal requirement is merely the stationarity of u_t.

Evans and Lewis (1995) noted that when π_t is used as a proxy for π_t^e, u_t may be highly persistent over small samples due to the 'peso problem'. Such high persistence in u_t, which enters into Equations (2) and (3) through $\pi_t^e = \pi_t - u_t$, can bias unit root tests toward finding non-stationarity in real interest rates and also distort the cointegrating regression. To minimize the possible effects of the 'peso problem', Evans and Lewis (1995) considered two other proxies for π_t^e: one consists of inflation forecasts produced by a Markov switching model, and the other is based on the Livingston Survey data on inflation expectations. Both proxies are shown to yield results more favourable to the long-run Fisher relationship than realized inflation.

In this study, following Darin and Hetzel (1995), inflation forecasts made by Data Resources Incorporated (DRI) are used as a proxy for π_t^e. These forecasts have been utilized by many corporations, financial institutions and government agencies. The DRI inflation forecasts are found to be broadly similar to those provided by the Greenbook, the Michigan Survey of Consumers, and the Livingston Survey. A possible criticism of using this type of proxies for π_t^e is that individual expectations are heterogeneous and that how prices aggregate individual expectations is far from clear. Darin and Hetzel (1995) observed that the broad similarity between the DRI inflation forecasts and those from the various other sources suggests these forecasts may reasonably capture the public's inflation expectations over the relevant sample period. Of course, like using any of the other proxies, the same maintained hypothesis remains assumed, namely, $\tilde{\pi}_t^e$ and π_t^e differ merely by stationary errors.

It should be noted that a direct measurement of expected inflation might be possible if Treasury securities were indexed and linked to the consumer price index (CPI) (Kandel *et al*., 1995). Expected inflation would then be measured by the difference in yields between indexed and non-indexed Treasury securities of the same maturity. For the US data examined in this study, nevertheless, no data for such indexed bonds are available.

3. DATA ON *EX ANTE* REAL RATES

The data under study are obtained from Darin and Hetzel (1995). Expected inflation is measured as CPI inflation forecasts, which are available from DRI on a monthly basis regularly since November 1973. Three monthly series of US real interest rates are examined: two are based on the 6-month and 1-year Treasury bill rates, and one is based on the 6-month commercial paper rate. First, the real rate for the 1-year Treasury bill is computed as the difference between the 1-year Treasury bill rate and the 4-quarter CPI inflation rate predicted by DRI. The resulted real rate series, covering the period from November 1973 through July 1994, yields 249 observations. Second, the real rate for the 6-month Treasury bill is obtained as the difference between the 6-month Treasury bill rate and 2-quarter expected inflation measured by the geometric average of the quarterly CPI inflation predictions. This real rate series extends over the same sample period as the 1-year bill rate series. Third, the real rate for the 6-month commercial paper is calculated as the difference between the 180-day commercial paper rate and 2-quarter expected inflation measured by the geometric average of the quarterly CPI inflation forecasts. The commercial paper real rate series starts from November 1973 to December 1994 and has 254 observations.

In calculating the monthly series of *ex ante* real rates, the date of the Treasury bill or commercial paper rate is chosen to match fairly closely the control date for the DRI inflation forecasts, which is the end of the preceding month. The DRI inflation forecasts are reported in monthly issues of *Review of the US Economy* published by DRI/McGraw-Hill. Details about the data sources and data construction are described by Darin and Hetzel (1995).

4. TESTING FOR A UNIT ROOT: I(1) OR I(0)?

All the series of real interest rates are first tested for a unit root using the standard augmented Dickey–Fuller (ADF) test. For a time series $\{y_t\}$, the ADF test involves the following regression:

$$\Delta y_t = \mu + \gamma y_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta y_{t-j} + v_t, \qquad (5)$$

where δ is the difference operator and v_t is a random error term. The null hypothesis of a unit root is represented by $\gamma = 0$. The ADF statistic is given by the usual t-statistic for the γ coefficient. Results of the ADF test are reported in Table 1. To allow for the potential sensitivity of the results to the lag choice, statistics for $p = 2$, 4 and 6 are reported. The results in general are at best mixed, providing not much support for mean reversion in the real interest rate. The I(1) hypothesis can be rejected in the case of the 6-month commercial paper only. Tests with a time trend have also been performed; they yield similar results. Since the time trend is not statistically significant for all the series, those results are not reported.

The empirical failure to reject the I(1) hypothesis does not definitively establish the presence of a unit root. The power of standard unit root tests, like the Dickey–Fuller-type tests, is known to be low against plausible stationary alternatives. An inherent problem arises from the specification of the null hypothesis. The unit root tests typically take nonstationarity as the null hypothesis, which cannot be rejected unless there is strong evidence against it.

To address the problem, Kwiatkowski *et al.* (1992) devised a Lagrange multiplier test, referred to as the KPSS test, under a null hypothesis of

Table 1. Results of the augmented Dickey–Fuller test for a unit root

Series	$p=2$	$p=4$	$p=6$
6-month Treasury bill	-2.524	-2.546	-2.031
6-month commercial paper	-2.635*	-2.768*	-2.132
1-year Treasury bill	-2.375	-2.364	-1.992

Notes: The ADF test examines the null hypothesis of an I(1) process against the alternative of an I(0) process. The p parameter is the number of lagged differences included in the ADF regression. Both the 6-month and 1-year Treasury bill series have 249 observations; whereas, the 6-month commercial paper series has 254 observations. Finite-sample critical values of the ADF test for a sample size of 250 are computed from response surface equations (Cheung and Lai, 1995a). The 5% and 10% critical values are given respectively by -2.862 and -2.566 for $p=2$; -2.856 and -2.561 for $p=4$; and -2.851 and -2.556 for $p=6$. Statistical significance is indicated by an asterisk (*) for the 10% level.

stationarity. The KPSS test considers that a time series can be written as the sum of a random walk and a stationary error, and that the variance of the error in the random walk component equals zero under the null hypothesis. To carry out the test, the residual series, e_t, is first obtained from a regression of y_t on a constant and possibly a trend. The KPSS statistic, denoted by $\hat{\eta}_\mu$, is constructed as

$$\hat{\eta}_\mu = T^{-2} \sum_{t=1}^{T} S_t^2 / s^2(l) \tag{6}$$

where S_t is the partial sum process of regression residuals defined by

$$S_t = \sum_{i=1}^{t} e_i, \qquad t = 1, 2, \cdots, T \tag{7}$$

and $s^2(l)$ is a heteroscedasticity and autocorrelation consistent variance estimator given by

$$s^2(l) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{j=1}^{l} \sum_{t=j+1}^{T} w(j, l) e_t e_{t-j}. \tag{8}$$

with l being a lag truncation parameter and $w(j, l) = 1 - j/(l+1)$, a weighting function corresponding to the choice of the Barlett window. Kwiatkowski *et al*. (1992) showed that this test for stationarity has good size and power properties in simulation analysis. The KPSS test is a one-sided upper tail test. When the KPSS statistic is too large, the null hypothesis of I(0) processes is rejected in favour of non-stationary alternatives of I(1) processes. Asymptotic critical values for the test are provided by Kwiatkowski *et al*. (1992, Table 1); they are found to be good approximations to finite-sample critical values in our case. The results of the KPSS test are presented in Table 2. The I(0) hypothesis is strongly rejected for every series of real interest rates under study.

Table 2. Results of the KPSS test for stationarity

Series	$l=2$	$l=4$	$l=6$
6-month Treasury bill	1.655**	1.039**	0.771**
6-month commercial paper	1.553**	0.978**	0.730**
1-year Treasury bill	1.570**	0.977**	0.721**

Notes: The KPSS test examines the null hypothesis of an I(0) process against the alternative of an I(1) process. l is the lag truncation parameter. Asymptotic critical values are given by 0.463 and 0.347 for the 5% and 10% levels of significance. Finite-sample critical values for $T=250$ are estimated based on the Monte Carlo method, using 30000 replications. The 5% and 10% critical values are given respectively by 0.462 and 0.350 for $l=2$, 0.458 and 0.348 for $l=4$, and 0.452 and 0.346 for $l=6$. The estimated standard errors for these estimates range from 0.003 to 0.005 for the 5% critical values and from 0.002 to 0.003 for the 10% ones. Statistical significance is indicated by a double asterisk (**) for the 5% level.

If a choice is restricted to be between I(0) and I(1) dynamics only, the KPSS test results, together with those of the ADF test, will indicate that the real rates for both the 6-month and 1-year Treasury bills follow an I(1) process. On the other hand, the empirical evidence for the 6-month commercial paper rate is ambiguous. The evidence suggests neither I(1) nor I(0) behaviour, pointing to the need to explore other possible alternatives.

The disparate findings can be reconciled and explained by considering fractionally integrated processes, which exhibit mean reversion but at a slow rate of hyperbolic decay. The ADF test for a unit root, which gives only tenuous evidence against I(1) behaviour, is known to have low power against fractionally integrated alternatives, whereas, the KPSS test for stationarity, which consistently detects non-I(0) dynamics, may have good power against alternatives of fractional integration. In an earlier study, Lai (1996) considered local AR alternatives and uncovered evidence of non-I(1) dynamics in real interest rates. This supplementary evidence, along with the non-I(0) findings here, further reinforces the need to go beyond the traditional I(1)/I(0) testing framework.

5. TESTING FOR FRACTIONAL INTEGRATION

An I(d) process has the following representation (Granger and Joyeux, 1980; Hosking, 1981):

$$B(L)(1-L)^d y_t = D(L)\varepsilon_t \tag{9}$$

where L is the lag operator, $B(L)$ and $D(L)$ are finite-order polynomials with stable roots, ε_t is white noise, and d can assume non-integer values. The fractional differencing operator, $(1-L)^d$, yields an infinite-order polynomial in L with slowly declining coefficients, given that

$$(1-L)^d = \sum_{k=0}^{\infty} \Gamma(k-d) L^k / \{\Gamma(k+1)\Gamma(-d)\} \tag{10}$$

where $\Gamma(\cdot)$ is the Gamma function. The I(d) process allows for a very rich class of spectral behaviour at low frequencies. Its spectral density, $f_y(\lambda)$, behaves like λ^{-2d} as $\lambda \to 0$. For $d > 0$, $f_y(\lambda)$ is unbounded at frequencies approaching 0, rather than bounded as for stationary autoregressive and moving average (ARMA) processes. By permitting d to take non-integer values, the fractional model can therefore capture a wide range of long-run, low-frequency behaviour not accommodated by traditional time-series models.

The long-term persistence of the y_t process is determined by its order of integration, d. Specifically, it depends on whether $d < 1$ or not. It is generally known that the effect of a shock persists forever for an I(1) process but dies out for an I(0) process. A shock-dissipating process does not have to be I(0) exactly, however. A more general I(d) process with $d < 1$ can also display shock dissipation. This can be seen from the moving average representation for $(1 - L)y_t$:

$$(1 - L)y_t = A(L)\varepsilon_t \tag{11}$$

where $A(L) = 1 + \alpha_1 L + \alpha_2 L^2 + \dots$, derived from

$$A(L) = (1 - L)^{1-d}\Phi(L) \tag{12}$$

for $\Phi(L) = B^{-1}(L)D(L)$. The moving average coefficients, α_is, are called the impulse responses. The impact of a unit innovation at time t on y_{t+k} equals $1 + \alpha_1 + \alpha_2 + \cdots + \alpha_k$. The infinite cumulative impulse response, $A(1)$, thus measures the long-run impact of the innovation. Cheung and Lai (1992, 1993) showed that for $d < 1$, $A(1) = 0$, implying shock-dissipating behaviour. For $d = 1$, $A(1) = \Phi(1) \neq 0$ and so the effect of a stock will not die out. For $d > 1$, $A(1) = \infty$ and there will be shock amplification, not dissipation. Accordingly, mean reversion (i.e. $A(1) = 0$) occurs only when $d < 1$, so a test for fractional integration can serve as a test for mean reversion.

A spectral regression-based procedure devised by Geweke and Porter-Hudak (1983) is used to test for fractional integration. This semi-nonparametric test, called the GPH test, has been applied by Cheung and Lai (1993, 1995b), Diebold and Rudebusch (1989, 1991b), and Shea (1991), among others. The test requires no exact parameterization of short-term ARMA dependency and is robust to conditional heteroscedasticity. Monte Carlo results reported by Cheung (1993a) indicate that the GPH test is robust to conditional heteroscedastic effects and moderate ARMA dependence, although not to large ARMA components. A more powerful method for estimating the integration order is the maximum likelihood (ML) procedure, as used by Baillie and Bollerslev (1994), Cheung (1993b), Cheung and Lai (1992), and Sowell (1992). Not much is known about the robustness of the ML procedure to data heterogeneity, however. Results of preliminary data examination indicate the presence of substantial conditional heteroscedasticity but small short-term dependency in first differences of real interest rates. Given its robustness to conditional heteroscedasticity, the GPH test is particularly appropriate for the present application. In addition, test power should not be a matter of concern here. According to our results, the GPH test performs well in detecting I(d) behaviour.

The GPH procedure involves estimation of the fractional integration order, d, using a simple spectral regression for the differenced series:

$$\ln(I(\lambda_j)) = \phi_0 - \phi_1 \ln(4\sin^2(\lambda_j/2)) + \zeta_t, \quad j = 1, 2, \dots, n, \tag{13}$$

where $I(\lambda_j)$ is the periodogram at harmonic frequency $\lambda_j = 2\pi j/T$, ζ_t is the random error, and $n = T^\mu$ for $0 < \mu < 1$ is the number of low-frequency ordinates used for the regression. The least squares estimate of ϕ_1 provides a consistent estimate of $1 - d$, and hypothesis testing regarding the value of d can be conducted based on the usual t-statistic. The number of low-frequency ordinates, n, used in the spectral regression is a choice variable. The choice involves judgement. If n is too large, the regression will lead to biased d estimates due to contamination caused by high-frequency dynamics. If n is too small, on the other hand, it will yield imprecise estimates due to limited degrees of freedom in estimation. To balance these two consideration factors, different μ values are used for the sample-size function, $n = T^\mu$. The results reported below are for $\mu = 0.60$, 0.625 and 0.65.

Table 3 contains estimates for the order of fractional integration, d, from the GPH spectral regression. The d estimates are reported along with their statistics for testing the null hypothesis of $d = 1$ against the alternative of $d < 1$ as well as for testing the null hypothesis of $d = 0$ against the alternative of $d > 0$. The GPH statistics are com-

Table 3. Results of fractional integration analysis

Series	μ	d	GPH statistic based on empirical error variance		GPH statistic based on theoretical error variance	
			H_0: $d=1$	H_0: $d=0$	H_0: $d=1$	H_0: $d=0$
6-month Treasury bill	0.600	0.702	−2.403**	5.663**	−1.981**	5.077**
	0.625	0.723	−2.502**	6.522**	−2.005**	5.227**
	0.650	0.727	−2.770**	7.381**	−2.157**	5.746**
6-month commercial paper	0.600	0.723	−2.205**	5.768**	−1.839**	4.812**
	0.625	0.761	−2.001**	6.357**	−1.732**	5.502**
	0.650	0.781	−2.046**	7.285**	−1.734**	6.172**
1-year Treasury bill	0.600	0.824	−1.525*	7.136**	−1.171	5.478**
	0.625	0.818	−1.774**	7.994**	−1.314*	5.918**
	0.650	0.834	−1.868**	9.413**	−1.309*	6.595**

Notes: The number of low-frequency ordinates used in the GPH spectral regression is determined by T^{μ}. The column beneath d gives the estimates of integration order. H_0: $d=1$ is tested against the alternative of $d<1$. H_0: $d=0$ is tested against the alternative of $d>0$. Asymptotic tests can be based on the standard t-distribution. Finite-sample critical values are given in Table 4. Statistical significance is indicated by an asterisk (*) for the 10% level and a double asterisk (**) for the 5% level.

Table 4. Finite-sample critical values for the Geweke–Porter-Hudak test

μ	GPH test based on empirical error variance		GPH test based on theoretical error variance	
	5% test	10% test	5% test	10% test
(A) Testing H_0: $d=1$ against H_a: $d<1$				
0.60	−1.805	−1.358	−1.722	−1.288
0.625	−1.783	−1.347	1.703	−1.286
0.65	−1.772	−1.343	−1.701	−1.303
(B) Testing H_0: $d=$ 0 against H_a: $d>0$				
0.60	1.627	1.277	1.564	1.238
0.625	1.625	1.267	1.573	1.236
0.65	1.622	1.267	1.578	1.250

Notes: Empirical distributions are obtained from 30,000 replications in each case.
(A) The data generating process used in the simulation is a random walk process. The null hypothesis H_0: $d=1$ is tested against the alternative H_a: $d<1$. Asymptotic critical values are given by −1.645 for the 5% test and −1.282 for the 10% test. The standard errors for the estimates of finite-sample critical values are computed, ranging from 0.011 to 0.019 for the 5% test and from 0.009 to 0.014 for the 10% test.
(B) The data generating process used in the simulation is a white noise process. The null hypothesis H_0: $d=0$ is tested against the alternative H_a: $d>0$. Asymptotic critical values are given by 1.645 for the 5% test and 1.282 for the 10% test. The standard errors for the estimates of finite-sample critical values are calculated, ranging from 0.009 to 0.013 for the 5% test and from 0.008 to 0.011 for the 10% test.

puted based on both the empirical error variance and the known theoretical error variance ($\pi^2/6$). The GPH test is derived from asymptotic results. To minimize the possible test bias in finite samples, finite-sample critical values for $T=250$ are estimated directly as quantiles of the empirical distributions obtained from simulations under the corresponding null hypotheses, using 30 000 replications. These estimates of critical values are tabulated in Table 4, and the computed standard errors for these estimates are satisfactorily small.

As reported in Table 3, the d estimates for real interest rates vary from 0.70 to 0.84. The GPH test results indicate that these d estimates are all significantly different from both 0 and 1, implying that the real interest rate is characterized by I(d) behaviour with $0<d<1$. Hence, the real interest rate contains not exactly a unit root but a fractional one. The evidence supports that the real interest rate exhibits mean reversion, though its dynamics can be rather persistent.

6. FURTHER EVIDENCE FROM *EX POST* REAL RATES

The analysis thus far has dealt with real interest rates that are measured using inflation forecasts exclusively. It is interesting to check whether the evidence of fractional integration holds up in real rates using actual inflation, on which much of the analysis in the literature has focused. Because of the given forecast horizons available for the DRI

Table 5. GPH test results for *ex post* real interest rates

Series	μ	d	GPH statistic based on empirical error variance H_0: $d=1$	GPH statistic based on empirical error variance H_0: $d=0$	GPH statistic based on theoretical error variance H_0: $d=1$	GPH statistic based on theoretical error variance H_0: $d=0$
1-month commercial paper	0.600	0.645	−2.344**	4.254**	−2.363**	4.288**
	0.625	0.567	−3.156**	4.126**	−3.135**	4.099**
	0.650	0.490	−3.949**	3.795**	−4.031**	3.875**
3-month Treasury bill	0.600	0.658	−2.171**	4.170**	−2.277**	4.374**
	0.625	0.583	−2.895**	4.039**	−3.020**	4.214**
	0.650	0.537	−3.554**	4.118**	−3.662**	4.244**
3-month commercial paper	0.600	0.624	−2.584**	4.295**	−2.498**	4.153**
	0.625	0.553	−3.328**	4.110**	−3.236**	3.997**
	0.650	0.503	−4.163**	4.212**	−3.930**	3.977**
6-month Treasury bill	0.600	0.636	−3.937**	6.875**	−2.422**	4.229**
	0.625	0.612	−4.412**	6.950**	−2.809**	4.425**
	0.650	0.716	−3.122**	7.886**	−2.242**	5.664**
6-month commercial paper	0.600	0.586	−3.513**	4.972**	−2.754**	3.897**
	0.625	0.566	−4.013**	5.231**	−3.141**	4.093*
	0.650	0.676	−2.890**	6.024**	−2.563**	5.343**
1-year Treasury bill	0.600	0.847	−1.235	6.837**	−1.017	5.634**
	0.625	0.875	−1.024	7.163**	−0.905	6.917**
	0.650	1.079	0.735	10.012**	0.627	8.533**

Notes: The number of low-frequency ordinates used in the GPH spectral regression is determined by T^{μ}. The column beneath d gives the estimates of integration order. H_0: $d=1$ is tested against the alternative of $d<1$. H_0: $d=0$ is tested against the alternative of $d>0$. Asymptotic tests can be based on the standard t-distribution. Finite-sample critical values are given in Table 4. Statistical significance is indicated by a double asterisk (**) for the 5% level.

inflation forecasts, the above analysis is limited to three series of *ex ante* real rates: rates on the 6-month and 1-year Treasury bills and the 6-month commercial paper. To allow for comparison with other studies, interest rates of shorter maturities are also examined for *ex post* real rates; they include real rates on the 1-month commercial paper, the 3-month commercial paper, and the 3-month Treasury bill. All the data on nominal rates and CPI inflation are obtained from the Federal Reserve Bank of St. Louis' *FRED* database. Altogether, six different series of *ex post* real rates are computed as the differences between the nominal rates on Treasury bills and commercial papers and the realized rates of inflation matching individual maturities. To maintain comparability, all these series cover a similar sample period as the data on *ex ante* real rates (November 1973 through December 1994).

The results of fractional integration analysis for *ex post* real interest rates are presented in Table 5. In five out of six cases can the hypotheses of $d=1$ and $d=0$ be both rejected in favour of the alternatives of $d<1$ and $d>0$ at the 5% level. The results suggest that the *ex post* real rate is characterized by fractional dynamics with mean reversion, as found in the *ex ante* real rate.

7. CONCLUSIONS

The long-term persistence of the real interest rate has been investigated. The long-run Fisher effect suggests that the nominal interest rate and expected inflation have a one-for-one equilibrium relationship. Accordingly, the *ex ante* real rate should be mean-reverting and not follow a non-stationary I(1) process. Prior studies in general find mixed and less than supportive evidence for mean reversion in real interest rates. These findings are commonly obtained from statistical analyses that maintain a knife-edged I(1)/I(0) distinction by considering integer orders of integration only. Such distinction appears unnecessarily strict because mean-reverting processes do not have to be I(0) series. More general fractionally integrated processes can capture a wider range of mean-reverting behaviour than I(0) processes. Using a test for stationarity, evidence is indeed found to support the claim that the real interest rate shows non-I(0) behaviour, confirming the need to depart from the stringent I(1)/I(0) testing framework.

This study explores the potential presence of a fractional unit root in the real interest rate. The study examines data on real interest rates constructed using both DRI inflation forecasts and realized inflation. Based on fractional integration analysis, considerable evidence is found to indicate that real interest rates display neither I(1) nor I(0) dynamics; instead, they follow an I(d) process with $0 < d < 1$. Hence, real interest rates contain not an exact but a fractional unit root. The findings of a fractional unit root are shown to be robust with respect to whether inflation forecasts or actual inflation rates are used to measure expected inflation. These findings imply that the dynamics of real interest rates, albeit quite persistent, exhibit mean reversion.

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