

Production Lags and Price Behaviour

By KON S. LAI and PETER PAULY

California State University and University of Toronto

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This paper explores the implications of the production lag for the firm's decisions. We establish a significant relationship between price behaviour and the length of the production lag. We show that specific results in the literature are crucially dependent upon the assumption about the production lag. By allowing the production time to exceed the decision period, the basic framework of earlier price-inventory models is significantly extended. The model thus incorporates finished-goods inventories as well as goods-in-process inventories.

INTRODUCTION

The literature on price-inventory models has notably ignored the effects of different production lags on firms' decisions. Production lags essentially determine the intertemporal structure of firms' decision problems. One may expect that they can affect the price and output behaviour of individual firms. For example, Carlson (1973) and Carlson and Wehrs (1974) observe that the production lag can significantly influence firms' output decisions, and an understanding of its effects is important for dynamic economic analyses.

Earlier price-inventory analyses typically assume that the production time does not exceed the decision period.¹ As Holt and Modigliani (1961) note, however, the production time may well be significantly longer than the decision period; this is a characteristic of many durable goods industries. Production lags are also critical in many models of economic fluctuations, e.g. the time-to-build model of Kydland and Prescott (1982) and the input-output model of Long and Plosser (1983). Furthermore, in assuming the production time not to exceed the decision period, previous inventory models recognize the existence only of finished-goods inventories. To allow these models to apply to a more general framework including goods-in-process inventories, it is necessary to extend the models to a more general production lag structure such that planned production will materialize not as finished goods but as goods in process by the end of a decision period. In this way, the extended model can account for the dynamics of goods-in-process inventories and their possible effects on the firm's decisions.

Little research has sought to explore the implications of the production lag for the firm's behaviour. This paper takes a first step in that direction. It proceeds as follows. Section I outlines a one-period production lag model. Section II extends the model to include multi-period production lags and examines the implications of the production lag for price and output behaviour. Section III concludes.

I. THE MODEL WITH A ONE-PERIOD PRODUCTION LAG

We consider a linear-quadratic model of a monopolistic firm, based on Blinder (1982). In each period t the firm faces a random demand schedule,

$$(1) \quad y_t = Q(p_t, u_t) = a - bp_t + u_t, \quad b > 0,$$

where y_t is the amount of sales, p_t is the price set and u_t is the demand shock, considered to follow a simple AR (1) process,

$$(2) \quad u_t = \tau u_{t-1} + e_t, \quad 0 \leq \tau \leq 1,$$

where e_t is white noise. Denote by x_t the production level in period t . The production cost function is assumed to be represented by

$$(3) \quad C(x_t, s_t) = c_0 + (c_1 + s_t)x_t + cx_t^2, \quad c > 0,$$

where s_t is a cost disturbance term, assumed to be white noise.

Denote by v_t the inventory level at the beginning of period t . In this model there is a one-period lead time in production; i.e. output in period t is not available for sale until period $t+1$. The familiar inventory balance equation is given by

$$(4) \quad v_{t+1} = v_t + x_t - y_t.$$

The inventory holding cost function is assumed to be convex and is given by

$$(5) \quad H(v_t) = h_0 + h_1 v_t + h v_t^2, \quad h > 0.$$

The firm makes plans at the beginning of period 0, given an initial inventory holding of v_0 , to maximize the sum of discounted expected profits

$$(6) \quad E_0 \sum_{t=0}^{\infty} (1+r)^{-t} [p_t Q(p_t, u_t) - C(x_t, s_t) - H(v_t)],$$

subject to restriction (4), where E_0 is the expectation operator conditional on the information set available at the beginning of period 0; r is the rate of interest. Under the certainty-equivalent formulation, only the conditional means of variables determine optimal behaviour. For notational simplicity e_0 is suppressed in the following analysis, as in Blinder (1982). The necessary conditions for optimality are given by

$$(7) \quad \phi_t = (1+r)\phi_{t-1} + 2hv_t + h_1,$$

$$(8) \quad x_t = (\phi_t - c_1)/2c,$$

$$(9) \quad p_t = [(a + u_t)/b + \phi_t]/2,$$

where ϕ_t is the costate variable for (4), subject to the transversality condition that $E_0(1+r)^{-T}[(y_T/b + cx_T)(1+r) + hv_{T+1}] = 0$ as $T \rightarrow \infty$. Combining (1), (4), (8) and (9) gives

$$(10) \quad v_{t+1} - v_t = [(1/c + b)\phi_t - (c_1/c + a + u_t)]/2.$$

To solve the model, combine (7) and (10) to yield²

$$(11) \quad \phi_{t+1} - [2 + r + h(1/c + b)]\phi_t + (1+r)\phi_{t-1} = -h(c_1/c + a + \tau^t u_0).$$

Its solution is readily given by

$$(12) \quad \phi_t = \bar{\phi} + A_1 z_1^t + A_2 z_2^t + B\tau^{t+1},$$

where $\bar{\phi} = (c_1 + ac)/(1 + bc)$; $B = -hu_0/[(z_1 - \tau)(z_2 - \tau)]$; A_1 and A_2 are constants; and z_1 is the smaller root and z_2 the larger root of the equation

$$(13) \quad f(z) = z^2 - [2 + r + h(1/c + b)]z + (1 + r) = 0,$$

such that $0 < z_1 < 1 < z_2$. To exclude the unstable solution, A_2 is chosen to be zero. Equation (12) then implies (see Appendix) that

$$(14) \quad \phi_0 = \bar{\phi} + [hz_2u_0/(z_2 - \tau) - 2h(v_0 + \bar{v})]/(z_2 - 1),$$

where \bar{v} is the stationary level of inventory of the model.³ It follows that

$$(15a) \quad \partial\phi_0/\partial u_0 = hz_2/[(z_2 - \tau)(z_2 - 1)] > 0,$$

$$(15b) \quad \partial\phi_0/\partial v_0 = -2h/(z_2 - 1) < 0,$$

since $h > 0$ and $z_2 > 1$. Conditions (15a) and (15b) immediately lead to Blinder's (1982) Propositions 1 and 4, restated respectively as follows.

Proposition 1. Price and production react positively and planned inventory investment reacts negatively to expected demand shocks.

Proof. From (8), (9), (10) and (15a), we have

$$(16a) \quad \partial p_0/\partial u_0 = (1/b + \partial\phi_0/\partial u_0)/2 > 0,$$

$$(16b) \quad \partial x_0/\partial u_0 = (\partial\phi_0/\partial u_0)/2c > 0,$$

$$(16c) \quad \partial I_0/\partial u_0 = [(\partial\phi_0/\partial u_0)(1/c + b) - 1]/2 < 0,$$

where $I_0 = v_1 - v_0$, the planned inventory investment in period 0. \square

Proposition 1*. Firms with higher beginning-of-period inventories will tend to produce less, set a lower price and reduce inventory investment.

Proof. From (8), (9), (10) and (15b), we obtain

$$(17a) \quad \partial p_0/\partial v_0 = (\partial\phi_0/\partial v_0)/2 < 0,$$

$$(17b) \quad \partial x_0/\partial v_0 = (\partial\phi_0/\partial v_0)/2c < 0,$$

$$(17c) \quad \partial I_0/\partial v_0 = (\partial\phi_0/\partial v_0)(1/c + b)/2 < 0. \quad \square$$

II. MULTI-PERIOD PRODUCTION LAGS

In this section we extend the model in Section I to allow for the decision period to be shorter than the production time. We first examine a model with a two-period production lag, the simplest model to capture the implications of production lags. Extension to higher-order production lags is then discussed.

(a) The two-period production lag case

In this analysis we will consider mainly a fixed production period. Holt and Modigliani (1961) note that the production time itself can be a variable, likely to increase as the rate of production rises. A general model with a variable production period, which can then take non-integer values, appears mathematically intractable, however. Moreover, empirical evidence presented by Carlson (1973) provides little support for Holt and Modigliani's assertion.

Because of a two-period production lead time, production in period $t - 1$ will not be available for sales until period $t + 1$. The inventory balance equation, accordingly, becomes

$$(18) \quad v_{t+1} = v_t + x_{t-1} - y_t.$$

For the purpose of comparison, we retain the specifications of the demand and cost functions as given by (1), (3) and (5). The problem of the firm is to maximize (6) subject to the constraint (18). The necessary conditions for optimality are then given by (7), (9), (18) and

$$(19) \quad x_t = [(1+r)^{-1}\phi_{t+1} - c_1]/2c.$$

Similar to equation (11), we obtain from (1), (7) and (9) that

$$(20) \quad \phi_{t+1} - [2+r+(1+r)^{-1}h/c+hb]\phi_t + (1+r)\phi_{t-1} = -h(c_1/c+a+\tau^t u_0).$$

Solving (20) straightforwardly as in Section I (see also Appendix) gives

$$(21a) \quad \partial\phi_0/\partial u_0 = hZ_2/\{(Z_2-\tau)[Z_2-1-(1+r)^{-1}h/c]\} > 0,$$

$$(21b) \quad \partial\phi_0/\partial v_0 = 2h/(Z_1-1-r-hb) < 0,$$

$$(21c) \quad \partial\phi_0/\partial x_{-1} = 2h/(Z_1-1-r-hb) < 0,$$

where $0 < Z_1 < 1 < Z_2$ and Z_1 and Z_2 are the roots of the equation

$$(22) \quad F(Z) = Z^2 - [2+r+(1+r)^{-1}h/c+hb]Z + (1+r) = 0.$$

Similarly, we can establish (see Appendix) that

$$(23a) \quad \partial\phi_1/\partial u_0 = -h(1+r)/[(Z_2-\tau)(Z_1-1-r-hb)] + h/(Z_2-\tau) > 0,$$

$$(23b) \quad \partial\phi_1/\partial v_0 = 2hZ_1/(Z_1-1-r-hb) < 0.$$

It follows from (9) that

$$(24a) \quad \partial p_0/\partial u_0 = (1/b + \partial\phi_0/\partial u_0)/2 > 0,$$

$$(24b) \quad \partial p_0/\partial v_0 = (\partial\phi_0/\partial v_0)/2 < 0,$$

and from (19) that

$$(25a) \quad \partial x_0/\partial u_0 = (\partial\phi_1/\partial u_0)/[2c(1+r)] > 0,$$

$$(25b) \quad \partial x_0/\partial v_0 = (\partial\phi_1/\partial v_0)/[2c(1+r)] < 0.$$

Conditions (24a), (24b), (25a) and (25b) are clearly consistent with Propositions 1 and 1*.

(b) The effect of changes in goods-in-process inventories

In the presence of a two-period production lag, last-period production x_{-1} will become goods in process at the beginning of the current period, and x_{-1} can thus be treated as stocks of goods-in-process inventories. Accordingly, we next examine the effects of changes in goods-in-process inventories.

Maccini and Rossana (1984) and Reagan and Sheehan (1985) find evidence that finished-goods inventory investment is positively related to goods-in-process inventories. This finding is consistent with our analysis.

Proposition 2. Price and production will respond negatively and finished-goods inventory investment will respond positively to changes in stocks of goods-in-process inventories.

Proof. Since $I_0 = x_{-1} - y_0$, $\partial I_0 / \partial x_{-1} = 1 + b(\partial \phi_0 / \partial x_{-1}) / 2$. Hence

$$\partial I_0 / \partial x_{-1} = (1 + r - Z_1) / (1 + r + hb - Z_1) > 0,$$

using (21c). Likewise, we can show that $\partial p_0 / \partial x_{-1} < 0$ and $\partial x_0 / \partial x_{-1} < 0$. \square

It should be noted that the model here has assumed that current goods in process are all used to produce finished goods for the next period. This simplification allows us to focus the analysis on the implications of production lags and to ignore decisions about the stocks of goods in process. The simple model still enables us to establish some insights into the relationships between the time lag and the firm's decisions. A more complete and less mechanical model would consider that goods in process can be stocked over several decision periods, and in that case a detailed specification of the benefits and costs of holding goods in process is required. Such further generalization of the model is of interest in future research.

(c) *The effect of the interest rate on price behaviour*

The Blinder model predicts that price is stickier as the rate of interest rises.⁴ We observe that this prediction may be altered because of production lags. Direct differentiation of (21a) with respect to r yields

$$(26) \quad \partial(\partial \phi_0 / \partial u_0) / \partial r = \eta(\partial Z_2 / \partial r) + \omega,$$

where

$$\eta = \{\tau[1 + (1+r)^{-1}h/c] - Z_2^2\} / \{(Z_2 - \tau)[Z_2 - 1 - (1+r)^{-1}h/c]\} < 0,$$

since $Z_2 > 1 + (1+r)^{-1}h/c > \tau$, and where

$$\omega = (\partial \phi_0 / \partial u_0) / [Z_2 - 1 - (1+r)^{-1}h/c] + \partial[h / (1+r)c] / \partial r < 0.$$

Equation (26) cannot be signed unambiguously unless $\partial Z_2 / \partial r > 0$. The result thus depends critically upon the magnitude of h/c . In the general case of $h > 0$, the effect of the interest rate is indeterminate. The point is that a rise in r has in part the effect of increasing the discount factor (and thus reducing the impacts of future demand), and in part an effect of raising the effective production cost in terms of the opportunity cost of time. While the former effect tends to reduce price responsiveness to demand changes, the latter effect tends to raise it.

(d) *Further implications of production lags*

Blinder's model suggests that a firm facing linear production costs would not change price even when stuck with huge inventories, and that the firm would fully correct any inventory disequilibrium within a period by adjusting current output only.⁵ As Abel (1985) notes, the latter implies that production-smoothing does not occur with linear production costs.

The consideration of production lags can eliminate these peculiar implications for price and output behaviour. From (21b) and (24b) we can establish that $\partial p_0 / \partial v_0 \rightarrow -h / (1+r+hb)$ as $c \rightarrow 0$. Hence changes in inventories will affect

price even if the production cost function is linear. In addition, combining (23b) and (25b) and simplifying yields

$$(27) \quad \partial x_0 / \partial v_0 = -[1 + r - Z_1(1 + r + hb)] / \{(1 + r)[1 + bc + (1 + r)^{-1}hb]\}.$$

Taking the limit, we have $\partial x_0 / \partial v_0 \rightarrow -(1 + r) / (1 + r + hb) > -1$ as $c \rightarrow 0$. It follows that current production responds only partially to inventory changes, even with linear production costs.

Abel suggests that, if demand is perfectly inelastic and if stockouts are considered, production-smoothing can occur even with a linear production cost function. The condition of perfectly inelastic demand seems restrictive. The analysis here indicates that, even with linear production costs and if stockouts are ignored, production-smoothing can occur as a result of production lags; but, in contrast to Abel's condition, if demand is perfectly price-inelastic, i.e. if $b = 0$, production-smoothing would not occur then (for $\partial x_0 / \partial v_0 = -1$). In the latter case, since sales are in effect given exogenously, the burden of adjustment entirely falls on production.

The results are in accord with intuition. When the production lag extends over more than one period, the flexibility of adjusting output is restricted. As a result, an inventory disequilibrium can no longer be corrected entirely within a period by adjusting production. It is then optimal for the firm to use price adjustments, in addition to production adjustments, to accommodate demand shocks. In the general case of non-zero c , indeed, the following proposition about price behaviour can be established.

Proposition 3. When production takes more than one decision period, price will become more responsive to changes in demand and finished-goods inventories.

Proof. Since $Z_2 < z_2$ and $Z_1 + Z_2 = 2 + r + hb + (1 + r)^{-1}h/c$, comparing (21a) and (21b) respectively with (15a) and (15b) indicates that $\partial \phi_0 / \partial u_0$ is more positive and $\partial \phi_0 / \partial v_0$ more negative with a two-period production lag than with a one-period production lag. The proposition then follows from (24a) and (24b). \square

It is reasonable to conjecture that firms with even longer production periods tend to display much stronger price responses to demand and inventory changes. Unfortunately, the dimensionality of the mathematical problem progressively increases and becomes unmanageable as the order of the production lag increases. A general result seems hard to derive. Nonetheless, in the next section we lay out the general structure of the solution for a k -period production lag case, and the explicit solution for the three-period production lag case is examined, in search of more support for our claim.

(e) Extension to higher-order production lags

Consider a general model with a k -period production lag ($k > 1$) such that production in period $t - 1$ will not be available for sales until period $t + k - 1$. To determine the comparative dynamic properties of this model, we have to solve for the value of ϕ_0 , as in our foregoing analysis. The general solution can be obtained by solving a system of recursive equations given as follows:

$$(28a) \quad \phi_{k-1} = \bar{\phi} + [\tau^{k-1} h \hat{Z}_2 u_0 / (\hat{Z}_2 - \tau) - 2h(v_{k-1} - \bar{v})] / (\hat{Z}_2 - 1),$$

and, for $j = 2, 3, \dots, k$,

$$(28b) \quad \phi_{j-1} = (1+r+hb)\phi_{j-2} + 2hx_{j-k-1} - h\tau^{j-2}u_0 + 2hv_{j-2} - ha + h_1,$$

$$(28c) \quad v_{j-1} = v_{j-2} + x_{j-k-1} - (a - b\phi_{j-2} + \tau^{j-2}u_0)/2,$$

where \hat{Z}_2 is the larger root of the equation

$$(29) \quad F(\hat{Z}) = \hat{Z} - [2+r+(1+r)^{-k+1}h/c + hb]\hat{Z} + (1+r) = 0.$$

Equation (28a) characterizes the optimal solution of the problem for period $k-1$ and beyond. For $k=1$, this reduces to (14) above. Equations (28b) and (29) are simply respective counterparts of (7) and (13) in the k -period lag model; (28c) describes the inventory dynamics of the model. According to (29), the larger k , the smaller the value of \hat{Z}_2 .

We have here a system of $2k-1$ recursive equations, which can be used to solve for $2k-1$ unknown variables in ϕ and v . In view of the complexity of the solution for the general case, we instead work out explicitly the comparative dynamics solution for $k=3$:⁶

$$(30a) \quad \partial\phi_0/\partial u_0 = [h\hat{Z}_2^2 + h(\hat{Z}_2 + \xi)]/[(\hat{Z}_2 - \tau)\xi(1+r+hb) + hb\hat{Z}_2(\hat{Z}_2 - \tau)],$$

$$(30b) \quad \partial\phi_0/\partial v_0 = -2h(\hat{Z}_2 + \xi)/[\xi(1+r+hb) + hb\hat{Z}_2],$$

$$(30c) \quad \partial\phi_0/\partial x_{-2} = -2h(\hat{Z}_2 + \xi)/[\xi(1+r+hb) + hb\hat{Z}_2],$$

$$(30d) \quad \partial\phi_0/\partial x_{-1} = -2h\hat{Z}_2/[\xi(1+r+hb) + hb\hat{Z}_2],$$

where $\xi = (1+r+hb)\hat{Z}_2 - 1 - r$. Based on these results, we can establish some properties of the present model.

In comparison to Proposition 3 regarding the degree of price stickiness, a more general proposition can be established here.

Proposition 3*. Firms with relatively longer production lags tend to respond more strongly in prices to changes in demand and inventories.

Proof. (21a) and (21b) can be rewritten as follows:

$$\partial\phi_0/\partial u_0 = hZ_2^2/\{(1+r+hb)Z_2 - 1 - r\}(Z_2 - \tau)\}$$

and

$$\partial\phi_0/\partial v_0 = -2hZ_2/[(1+r+hb)Z_2 - 1 - r],$$

since $Z_1Z_2 = 1+r$. Compare these results respectively with (30a) and (30b). Since $\hat{Z}_2(\hat{Z}_2 - 1) > (1+r+hb)\hat{Z}_2 - (1+r)$ and $Z_2 > \hat{Z}_2$, $\partial\phi_0/\partial u_0$ and $\partial\phi_0/\partial v_0$ are larger in absolute values with a longer production lag. The proposition then follows. \square

The basic point is that, with a longer production lag, more of the current adjustments have to be borne by a change in sales and hence by a change in price rather than a change in output.

When the production time exceeds two decision periods, the firm faces stocks of goods-in-process inventories at different stages of completion. Specifically, in the current decision period, x_{-1} represents goods in process at an earlier stage of completion than x_{-2} . We observe that price will react differently to changes in goods in process at different stages of completion.

Proposition 4. Price is less responsive to changes in goods-in-process inventories at an earlier stage of completion than that at a later stage.

Proof. The proposition follows directly from the fact that $|\partial\phi_0/\partial x_{-2}|$ is larger than $|\partial\phi_0/\partial x_{-1}|$, in view of (30c) and (30d). \square

Hence an increase in inventories at a later stage of completion is more 'alarming' than that still at an earlier stage of completion. The intuition is that the former will turn into finished goods awaiting sale sooner than the latter; consequently, the former creates greater incentives for the firm to stimulate sales than the latter.

III. CONCLUSIONS

In this paper we have established some significant relationships between the length of the production lag and the price and output adjustment behaviour. Since production lags can well vary across firms and commodities, this study may offer some guidance in explaining price and output behaviour in cross-sectional studies.

Our results also corroborate some of the earlier results in Abel (1985), Blinder (1982) and Carlson and Wehrs (1974) but modify them in a number of important aspects. First, while the Blinder model predicts that price is stickier as the interest rate rises, the prediction may be altered when production takes more than one period, and in that case the interest rate effect on price stickiness is generally indeterminate. Second, the consideration of production lags can resolve the paradox in Blinder's model that linear production costs make optimal pricing behaviour entirely insensitive to demand shocks and production smoothing will not occur, as noted by Abel. Finally, firms with longer production lags will exhibit stronger price responses to changes in demand and inventories than those with shorter production lags, other things being equal.

APPENDIX

(a) *To establish relationship (14)*

From equation (7) we have $\phi_1 = (1+r)\phi_0 + 2hv_1 + h_1$. In addition, (10) implies that $v_1 - v_0 = [(1/c + b)\phi_0 - (c_1/c + a + u_0)]/2$. To determine the value for A_1 , use these two conditions to substitute for ϕ_1 into the conditions $\phi_0 = \bar{\phi} + A_1 + B\tau$ and $\phi_1 = \bar{\phi} + A_1z_1 + B\tau^2$. We then obtain

$$A_1 = [-(h_1 + r\bar{\phi}) + B(\tau - z_1z_2) - 2hv_0]/(z_2 - 1).$$

Substituting for values of $\bar{\phi}$ and B and that of A_1 into (12), we obtain (14).

(b) *To establish conditions (21a), (21b) and (21c)*

The procedure is basically analogous to that derived above, except we now have $v_1 = v_0 + x_{-1} - (a - b\phi_0 + u_0)/2$ because x_{-1} is not a choice variable in period 0. From (7), this implies that

$$\phi_1 = (1+r+bh)\phi_0 + 2h(v_0 + x_{-1}) - hu_0 - ha + h_1.$$

Substituting for ϕ_1 as in (a) above and then simplifying, using the fact that

$$Z_1 - (1+r+bh) = Z_2 - 1 - (1+r)^{-1}h/c,$$

we can determine the value for A_1 . Then ϕ_0 is given by $\phi_0 = \bar{\phi} + A_1 + B\tau$.

(c) To establish conditions (23a) and (23b)

Using the fact that $v_1 = v_0 + x_{-1} - y_0$ and (7), we obtain that $\partial\phi_1/\partial u_0 = (1+r+hb)(\partial\phi_0/\partial u_0) - h$ and $\partial\phi_1/\partial v_0 = (1+r+hb)(\partial\phi_0/\partial v_0) + 2h$. Simplifying then yields (23a) and (23b).

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NOTES

1. See e.g. Abel (1985), Amihud and Mendelson (1983), Ashley and Orr (1985), Carlson (1986), Blinder (1982), Hay (1970) and Reagan (1982).
2. The solution strategy follows Blinder's analysis. This method is convenient in deriving the comparative dynamics of the current-period optimal decisions. We are interested in the current-period decisions, since planned decisions for the future can be revised later with revised forecasts of demand changes. An alternative method is to solve a difference equation in terms of the inventory variable (see e.g. Carlson 1986). This method is useful when the stochastic properties of the whole planned path for decision variables are examined and a rather general demand process is considered.
3. Schutte (1983) is concerned with the negativity of the stationary equilibrium level of inventory. In our analysis the non-stochastic stationary level of inventory \bar{v} is given by $2h\bar{v} = -h_1 - r(c_1/c + a)/(1/c + b)$ in view of (7) and (13). As Zabel (1986) has noted, h_1 can be negative when backlogging is more costly than storage. In that case, \bar{v} can be positive.
4. It is straightforward to show from (15a) that $\partial(\partial\phi_0/\partial u_0)/\partial r < 0$ for $z_2 > \tau$ and $\partial Z_2/\partial r > 0$. The result then follows from (16a).
5. Since, as $c \rightarrow 0$, $z_2 \rightarrow \infty$ and $z_1 \rightarrow 0$, it then follows from (16a), (16b) and (16c) that as $c \rightarrow 0$, $\partial p_0/\partial v_0 \rightarrow 0$ and $\partial x_0/\partial v_0 \rightarrow -1$.
6. The solution for the case of $k = 3$ is derived by substituting (28c) for v_1 and v_2 into (28a), and then substituting that for ϕ_1 and ϕ_2 into (28b).

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