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Is the real interest rate unstable? Some new evidence

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Prior studies typically report that real Treasury bill returns have a unit root. The unit-root findings are not consistent with the long-run Fisher effect and consumption-based asset pricing models. This study examines a data set of *ex ante* real returns on US Treasury bills and commercial papers. The statistical analysis employs a new modified Dickey–Fuller test, which has better power than standard unit-root tests. In contrast to previous findings, strong evidence of stationarity is found for all the real return series under examination. Implications of the results are discussed.

I. INTRODUCTION

Economists constantly seek a better understanding of the dynamic behaviour of the *ex ante* real interest rate. The *ex ante* real interest rate is an important variable influencing investment and output decisions. It can crucially affect valuations of financial assets and influence macroeconomic dynamics. More generally, it is a key variable common in virtually all intertemporal models. Rose (1988) raised an issue concerning the potential instability of the *ex ante* real interest rate and its implications for standard intertemporal asset pricing models.

Without examining data on *ex ante* real interest rates directly, Rose (1988) evaluated the orders of integration of inflation rates and nominal interest rates. Using conventional unit-root tests, the nominal interest rate is shown to have a unit root but not the inflation rate. The nominal interest rate can be written as the sum of the inflation rate, inflation-forecast errors, and the *ex ante* real interest rate. Under the condition of stationary inflation-forecast errors, the results represent indirect evidence that the *ex ante* real interest rate displays unit-root nonstationarity (see also King and Watson (1992) and Mishkin (1992) for similar nonstationarity findings). The presence of a unit root has, indeed, often been imposed in time-series modelling of real interest rates (e.g., Antoncic (1986), Fama and Gibbons (1982), and Garbade and Wachtel (1978)).

Rose (1988) observed that the presence of a unit root in the real interest rate is inconsistent with Lucas-type consumption-based asset pricing models (Breeden (1979), Hansen and Singleton (1982, 1983), and Lucas (1978)). According to the consumption-based models, investors adjust their consumption and investment plans over time such that the expected discounted value of the returns on an asset will have an equilibrium relationship with some marginal utility ratios of current to future consumption. As a result, these models predict that the time-series properties of asset returns are closely related to those of consumption growth. Rose (1988) examined stationarity in consumption growth processes and found that the growth rate of consumption contains no unit root. Given the findings that the consumption growth rate does not have a unit root but the real interest rate does, the empirical validity of consumption-based asset pricing models has been called into question.

The interpretation of Rose's (1988) results is far from definite, nevertheless. Choi (1994) noted that when inflation forecast errors are large relative to the variation of nominal interest rates, statistical findings of stationary inflation rates can be misleading and do not necessarily imply nonstationarity in real interest rates. If inflation really has a unit root, on the other hand, it is then possible for the real interest rate to be stationary. In contrast to Rose (1988), recent studies by, e.g., Bonham (1991), Jacques (1995), Mishkin (1992), and Wallace and Warner (1993) reported that inflation contains a unit root.

Another issue concerns the long-run relationship between nominal interest rates and expected inflation, implied by the long-run superneutrality of money (e.g., Evans and Lewis (1995), King and Watson (1992), MacDonald and Murphy (1989), and Mishkin (1992, 1995)). According to the Fisher (1930) relationship, nominal interest rates and expected inflation move together one-for-one in the long run. For the

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long-run Fisher relationship to hold, the *ex ante* real rate – the difference between the nominal rate and expected inflation – should display mean reversion. Finding a unit root in the real rate thus casts doubt on the relevance of the long-run Fisher relationship.

The empirical failure to reject the unit-root hypothesis should be interpreted with caution, however. The non-rejection does not lead to outright acceptance of non-stationarity, but rather it may simply reflect the low power of conventional unit-root tests to detect stationarity. Moreover, the unit-root finding for the real interest rate is based on *ex post* inflation data. The market might anticipate possible shifts in the inflation process, which did not materialize and show up in actual inflation data – a point noted by Evans and Lewis (1995), who argued that the use of realized inflation data can induce nonstationarity in measuring real interest rates.

This study re-examines the issue concerning the non-stationarity of the real interest rate by providing evidence directly based on a set of *ex ante* real interest rate data and by employing an improved Dickey–Fuller test proposed by Elliott, Rothenberg, and Stock (1992). The data under study are collected and constructed by Darin and Hetzel (1995) using inflation forecasts provided independently by the Green Book and Data Resources Incorporated (DRI). The Green Book is a document prepared by the staff of the Board of Governors of the Federal Reserve System and circulated prior to Federal Open Market Committee (FOMC) meetings. The DRI forecasts are made by professional forecasters and reported in monthly issues of *Review of the US Economy* published by DRI/McGraw-Hill. Both sources of inflation forecasts have been followed and utilized by many corporations, financial institutions, and government agencies. Darin and Hetzel (1995) observed that the Green Book and DRI forecasts generally move together and that broad similarities are also found between these forecasts and other inflation predictions from different sources such as the Livingston Survey and Michigan Survey. Furthermore, the present analysis employs a new modified Dickey–Fuller test, which has better power than standard unit-root tests. The improvement in test power achieved by the modified unit-root test is found to be important for a proper evaluation of the stationarity property of the real interest rate.

II. A STANDARD CONSUMPTION-BASED ASSET PRICING MODEL

Consumption-based asset pricing models suggest that the equilibrium price of an asset is determined by the expected present value of its future returns, adjusted by the intertemporal marginal rate of substitution in consumption. Consider a representative agent who chooses consumption and investment plans so as to maximize the expected value of a time-additive utility function:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right], \quad 0 < \beta < 1 \tag{1}$$

where β is a time preference parameter, c_t is consumption at time t, and E_t denotes expectations conditioned on information available at t. The utility maximization is under the budget constraint

$$c_t + p_t \cdot w_{t+1} \leqslant (p_t + d_t) \cdot w_{t+1} \tag{2}$$

where w_t is the holdings of assets at time t, p_t is the vector of prices corresponding to w_t (net of any distributions during the period), and d_t denotes the vector of values of the distributions.

The first-order optimality condition (Lucas (1978)) is given by

$$E_t[\beta(U'(c_{t+1})/U'(c_t))R_{it+1}] = 1 \quad \forall i \tag{3}$$

where U' is the marginal utility of consumption and $R_{it+1} = (p_{it+1} + d_{it+1})/p_{it}$ is the real return (or, more precisely, one plus the real rate of return) on asset i in the holdings. For the utility function, $U(c) = c^{1-\alpha}/(1-\alpha)$ with $\alpha > 0$, which exhibits constant relative risk aversion (Merton (1973) and Rubinstein (1976)), the above first-order condition can be simplified to

$$E_t[\beta(c_{t+1}/c_t)^{-\alpha} R_{it+1}] = 1 \tag{4}$$

Following Hansen and Singleton (1983), an empirical representation of this equation typically considered in the literature assumes a log-linear form

$$\ln \beta - \alpha E_t[\ln(c_{t+1}/c_t)] + E_t[\ln R_{it+1}] = 0 \tag{5}$$

This suggests that the consumption growth rate and real asset returns should share similar time-series properties. Rose (1988) presented evidence that consumption growth contains no unit root but short-term real bill returns have one, a result at odds with Equation (5). Given that consumption growth has no unit root, whether real asset returns contain a unit root bears upon the empirical relevance of consumption-based asset pricing models.

III. DATA

The data under examination consist of five series of US real returns on fixed-income securities: two of them are constructed based on commercial paper rates and the other three are based on Treasury bill (T-bill) rates. Expected inflation is measured alternatively as the DRI forecasts of consumer price index inflation and the Green Book's predictions of implicit output deflator inflation.

Table 1 contains a general description of the data series. The Green Book return series are labelled as G1 and G2, whereas the DRI return series are given by D1, D2, and D3.

Table 1. *Description of data series*

Series label	Sample period[a]	Sample size	Data description
G1	11/1965 to 1/1979	159	One- to two-quarter commercial paper (Green Book real returns)
D1	11/1973 to 12/1994	254	Two-quarter commercial paper (DRI real returns)
G2	11/1965 to 1/1979	159	One- to two-quarter Treasury bill (Green Book real returns)
D2	11/1973 to 7/1994	249	Two-quarter Treasury bill (DRI real returns)
D3	11/1973 to 7/1994	249	One-year Treasury bill (DRI real returns)

The Table contains a general description of the individual data series under study, including the series label, the sample period covered, the sample length, and the type of return data examined.
[a] The data are monthly return series, constructed as one plus the real rate of interest. The real interest rate is the nominal interest rate adjusted for expectred inflation. Data of longer sample periods have been provided by Darin and Hetzel (1995) but not on a regular monthly basis. For example, staring in early 1979, the FOMC met less than 12 times per year, making the observations of the Green Book real return series less frequently than monthly. The sample period considered here is chosen to avoid any problem of missing observations.

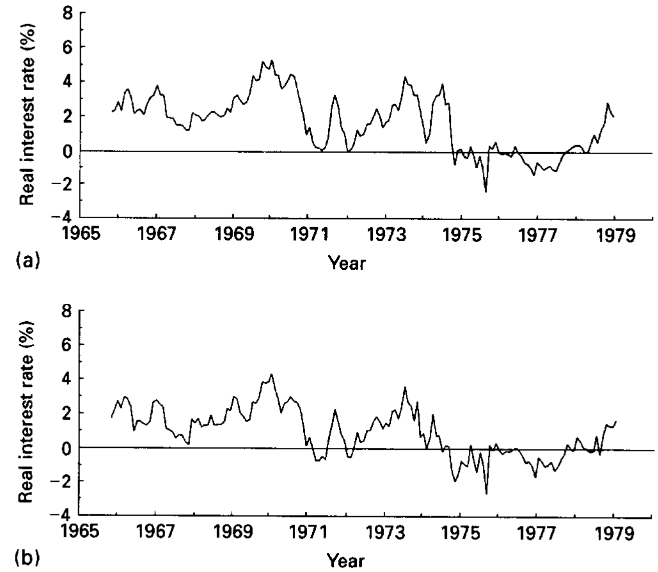

Fig. 1. *Plots of Green Book series of real interest rates*: (*a*) *1- to 2-quarter commercial paper;* (*b*) *1- to 2-quarter T-Bill*

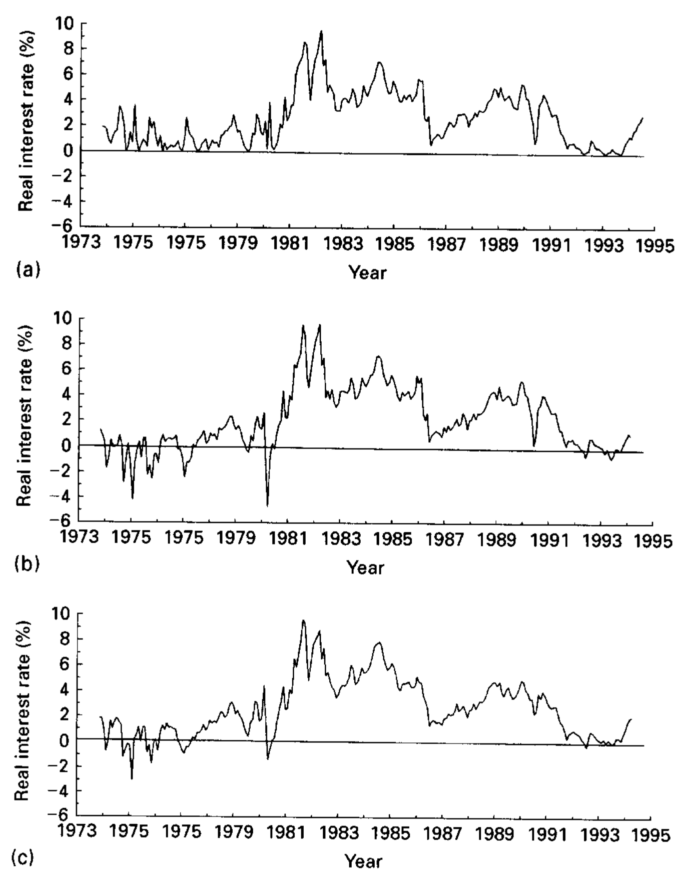

Fig. 2. *Plots of DRI series of real interest rates*; (*a*) *2-quarter commercial paper;* (*b*) *2-quarter T-Bill;* (*c*) *1-year T-Bill*

The real return is constructed as one plus the real rate of return. For the G1 series, the real rate of return on the one- to two-quarter commercial paper is computed as the difference between the three- to six-month commercial paper rate and two-quarter expected inflation measured by the weighted-average of the Green Book quarterly inflation forecasts. The G2 series is the real return on the one- to two-quarter T-bill, calculated like the G1 series except that the T-bill rate and not the commercial paper rate is considered. For the D1 series, the real rate of return on the two-quarter commercial paper is the difference between the 180-day commercial paper yield and two-quarter expected inflation measured by the geometric average of the DRI quarterly inflation forecasts. For the D2 series, the two-quarter T-bill real rate is given by the difference between the six-month T-bill yield and two-quarter expected inflation measured by the geometric average of the DRI quarterly inflation predictions. For the D3 series, the real rate of return on the one-year T-bill is computed as the difference between the one-year T-bill yield and the four-quarter inflation rate predicted by DRI. Plots of the Green Book and DRI series of real interest rates are given in Figures 1 and 2, respectively. Details about the data sources and data construction are described by Darin and Hetzel (1995).

IV. A MODIFIED DICKEY–FULLER TEST

The augmented Dickey–Fuller, or ADF, test (Dickey and Fuller (1979)) is often applied in empirical work to test for a unit root. Although it has been widely used, this standard unit-root test is known to have low power to find stationarity even when the series is actually stationary.

Elliott, Rothenberg, and Stock (1992) obtain the asymptotic power envelope for various unit-root tests by analysing the sequence of Neyman–Pearson tests of the unit-root null hypothesis ($\alpha = 1$) against the local alternative of $\bar{\alpha} = 1 + \bar{c}/T$ in an autoregressive (AR($p + 1$)) model, for which T is the sample size and $\bar{c}$ is a fixed constant. Based on asymptotic power calculation, it is shown that a modified Dickey–Fuller test, called the DF-GLS test, can achieve good gains in power over conventional unit-root tests.

The DF-GLS test is developed by considering asymptotic versions of optimal tests under local alternatives of $\bar{\alpha} = 1 + \bar{c}/T$, where $\bar{c} < 0$. Elliott, Rothenberg and Stock show that efficient estimation of an AR root can be achieved using demeaned and possibly detrended data, which are constructed as the residuals from a generalized least squares (GLS) regression. Under the local alternative $\alpha = \bar{\alpha}$, rather than under the null, the GLS regression is carried out based on the $(1 - \bar{\alpha}L)$ transformation of the data, with L being the lag operator. The locally GLS-demeaned or GLS-detrended data are then evaluated using Dickey–Fuller-type tests. The DF-GLS test is attractive in that it has much higher power than standard unit-root tests and that it is not difficult to implement, entailing one extra step only when compared with the ADF test.

Let $\{y_t\}$ be the data process under examination. The DF-GLS$^\tau$ test that allows for a linear time trend is conducted based on the following regression

$$(1 - L)y_t^\tau = a_0 y_{t-1}^\tau + \sum_{j=1}^{p} a_j (1 - L) y_{t-j}^\tau + u_t \qquad (6)$$

where u_t is a white noise error term; and y_t^τ, the locally detrended data process under the local alternative $\bar{\alpha}$, is given by

$$y_t^\tau = y_t - z_t \tilde{\beta} \qquad (7)$$

with $z_t = (1, t)$ and $\tilde{\beta}$ being the regression coefficient of $\tilde{y}_t$ on $\tilde{z}_t$, for which $\tilde{y}_t = (y_1, (1 - \bar{\alpha}L)y_2, \ldots, (1 - \bar{\alpha}L)y_T)'$ and $\tilde{z}_t = (z_1, (1 - \bar{\alpha}L)z_2, \ldots, (1 - \bar{\alpha}L)z_T)'$. The DF-GLS$^\tau$ test statistic is given by the usual t-statistic testing $a_0 = 0$ against the alternative of $a_0 < 0$ in regression (6). Elliott, Rothenberg and Stock recommend that the parameter $\bar{c}$, which defines the local alternative through $\bar{\alpha} = 1 + \bar{c}/T$, be set equal to -13.5. For the test without a time trend, denoted by DF-GLS$^\mu$, it involves the same procedure as the DF-GLS$^\tau$ test, except that y_t^τ is replaced with the locally demeaned series y_t^μ and also $z_t = 1$. In this case, the use of $\bar{c} = -7$ is recommended. The DF-GLS$^\mu$ test shares the same limiting distribution as the usual ADF test in the no-deterministic case.

V. EMPIRICAL ANALYSIS AND RESULTS

For comparison purposes, both ADF and DF-GLS tests for a unit root are performed on each of the T-bill or commercial paper return series. Real return series with and without the logarithm transformation are examined. Since the time trend is generally found to be insignificant, as noted by Rose (1988), results for unit-root tests with no time trend are reported below. In implementing the standard or modified Dickey–Fuller test, a choice of the lag order, denoted by p, is needed. The usual Akaike and Schwarz information criteria are applied to select p; both criteria consistently choose $p = 1$ in all the cases under examination. To check the potential sensitivity of the results to the lag choice, nonetheless, results corresponding to $p = 1, 2$, and 3 are obtained in each case.

The standard and modified Dickey–Fuller tests are all derived based on asyumptotic results. However, empirical applications necessarily deal with data of finite sample sizes. To minimize possible finite-sample bias for either test, finite-sample critical values for the ADF test are obtained from Cheung and Lai (1995a) and those for the DF-GLS test are from Cheung and Lai (1995b). For a given sample size and a given lag order, appropriate finite-sample critical values can be computed directly from simple response surface equations.

Table 2 contains the ADF test results along with the relevant finite-sample critical values for individual series. The results are mixed at best and not too supportive of the stationarity hypothesis. In three out of the five cases under consideration, the null hypothesis of a unit root cannot be rejected at the 5% significance level by the ADF test with any lag order. In only two cases, those of the D1 and D2 series, significant evidence rejecting the unit-root hypothesis can be found from the ADF test with $p = 1$. In these two cases, the results can be sensitive to the lag order. Whether the logarithm data transformation is used or not does not alter the results. These mixed results may be a symptom of the low power of the ADF test – the test applied by Rose (1988).

The results of the DF-GLS test are presented in Table 3. The DF-GLS statistics are reported together with their corresponding finite-sample critical values. These test results are in sharp contrast with those based on the ADF test, illustrating the usefulness of the modified Dickey–Fuller test. For all the real return series for T-bills and commercial papers, the unit-root hypothesis can be uniformly rejected at the 5% significance level. Both the logarithms and levels of the real return series yield qualitatively the same results. These results are also robust to the choice of lag order. In

Table 2. *Results of the standard Dickey–Fuller test*

Series	Sample size	Lag p	ADF statistic R_t	ADF statistic $\ln R_t$	Finite-sample critical value (5% test)
Commercial paper return series[a]					
G1	159	1	– 2.699	– 2.695	– 2.870
		2	– 2.690	– 2.677	– 2.866
		3	– 2.653	– 2.647	– 2.861
D1	254	1	– 3.145[b]	– 3.157[b]	– 2.865
		2	– 2.635	– 2.622	– 2.862
		3	– 2.860[b]	– 2.858	– 2.859
T-Bill return series[a]					
G2	159	1	– 2.837	– 2.839	– 2.870
		2	– 2.696	– 2.687	– 2.866
		3	– 2.793	– 2.790	– 2.861
D2	249	1	– 3.012[b]	– 3.034[b]	– 2.865
		2	– 2.524	– 2.509	– 2.862
		3	– 2.569	– 2.570	– 2.859
D3	249	1	– 2.649	– 2.647	– 2.865
		2	– 2.375	– 2.365	– 2.862
		3	– 2.384	– 2.386	– 2.859

The augmented Dickey–Fuller test is performed on the real return series with and without the logarithm transformation (R_t and $\ln R_t$). The test examines the null hypothesis of a unit root against the alternative of no unit root. Finite-sample critical values for the corresponding sample size and lag order are computed using the response surface equations estimated by Cheung and Lai (1995b).
[a] See Table 1 for a description of individual return series.
[b] The unit-root hypothesis can be rejected at the 5% level.

Table 3. *Results of the modified Dickey–Fuller test*

Series	Sample size	Lag p	DF-GLS statistic R_t	DF-GLS statistic $\ln R_t$	Finite-sample critical value (5% test)
Commercial paper return series[a]					
G1	159	1	– 2.599[b]	– 2.591[b]	– 2.051
		2	– 2.566[b]	– 2.549[b]	– 2.045
		3	– 2.544[b]	– 2.534[b]	– 2.039
D1	254	1	– 3.079[b]	– 3.097[b]	– 2.013
		2	– 2.572[b]	– 2.564[b]	– 2.010
		3	– 2.754[b]	– 2.760[b]	– 2.006
T-Bill return series[a]					
G2	159	1	– 2.593[b]	– 2.591[b]	– 2.051
		2	– 2.417[b]	– 2.404[b]	– 2.045
		3	– 2.504[b]	– 2.479[b]	– 2.039
D2	249	1	– 2.844[b]	– 3.872[b]	– 2.015
		2	– 2.368[b]	– 2.360[b]	– 2.011
		3	– 2.354[b]	– 2.361[b]	– 2.008
D3	249	1	– 2.599[b]	– 2.495[b]	– 2.015
		2	– 2.566[b]	– 2.204[b]	– 2.011
		3	– 2.544[b]	– 2.164[b]	– 2.008

The modified Dickey–Fuller (DF-GLS) test is performed on the real return series with and without the logarithm transformation (R_t and $\ln R_t$). The test examines the null hypothesis of a unit root against the alternative of no unit root. Finite-sample critical values for the corresponding sample size and lag order are computed using the response surface equations estimated by Cheung and Lai (1995a).
[a] See Table 1 for a description of individual return series.
[b] The unit-root hypothesis can be rejected at the 5% level.

sum, the results of the modified Dickey–Fuller test provide a much wider and stronger support for stationarity than those of the standard Dickey–Fuller test.

VI. CONCLUSION

The stationarity property of real returns on fixed-income securities has been investigated. Prior studies often suggest that the real bill return has a unit root. The findings conflict with usual consumption-based asset pricing models, since consumption growth has been found to contain no unit root. The unit-root result is, on the other hand, consistent with the random-walk model of real interest rates often considered in empirical studies (e.g., Antoncic (1986), Fama and Gibbons (1982), Garbade and Wachtel (1978)). Rose (1988) referred to such inconsistency between theoretical asset pricing models and the empirical behaviour of real asset returns as a 'puzzling' fact. The presence of a unit root in the real interest rate also calls into question the empirical relevance of the long-run Fisher relationship.

This study re-examines the issue and explores a newly available data set of *ex ante* real returns on T-bills and commercial papers. The analysis employs a new modified Dickey–Fuller test, which has better-power-than-usual unit-root tests. The improvement in test power is shown to be important for evaluating the stationarity property of the real interest rate. Empirical results strongly reject the unit-root hypothesis in favour of a stationary process for all the real return series under study. In finding stationary in real returns on T-bills and commercial papers, the results here provide no empirical support for the oft-used empirical random-walk model but resolve the 'puzzling' inconsistency concerning the behaviour of *ex ante* real returns implied by standard intertemporal asset pricing models. The stationarity finding also implies that deviations between the nominal interest rate and expected inflation are mean-reverting, in accordance with the long-run Fisher relationship.

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