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Nonlinear trend stationarity in global and hemispheric temperatures

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ABSTRACT
This study investigates the issue of whether our Earth's surface temperature exhibits a stochastic trend. Using state-of-the-art econometrics, we analyse the latest available temperature anomaly data. Our results indicate that both global and hemispheric temperatures may appear to have a stochastic trend when in fact they are stationary around a nonlinear deterministic trend and structural breaks are responsible. Furthermore, the nonlinearity found in the temperature trend is more complex than what has been reported in previous studies.

KEYWORDS
Stochastic trend; trend breaks; stationarity; surface temperature; global warming

JEL CLASSIFICATION
C22; Q54

I. Introduction

With the Earth's temperatures continuing to hit fresh highs, the subject of global warming has been at the forefront of social and political discussions. While the warming trend has been well noted, how the trend dynamics should be modelled statistically remains a hotly contested issue. This issue has an important bearing on climate modelling and the proper econometric methods for evaluating the contributing factors to the temperature trend. Kaufmann et al. (2010, 2013), for example, maintain that surface temperature is nonstationary and displays a stochastic trend. They support using cointegration techniques to analyse the relation between surface temperature and radiative forcing. Estrada and Perron (2016) observe, however, that the warming trend has not been steady, especially during the twentieth century. Once a one-time trend slope change is accounted for, the stochastic trend hypothesis can be rejected in favour of stationarity around a nonlinear trend. Mills (2013) also allows for a trend shift but finds at best mixed evidence on stationarity.

This study investigates the issue of whether a stochastic trend exists in surface temperature using state-of-the-art econometrics. Without restricting the analysis to a single break, we employ the minimum Dickey–Fuller (MDF) test devised by Harvey, Leybourne, and Taylor (2013). Along a similar line of research of Carrion-i-Silvestre, Kim, and Perron (2009), the MDF test implements generalized least squares (GLS) detrending and permits multiple breaks under both the null and the alternative hypotheses. In contrast to the analysis of Carrion-i-Silvestre, Kim, and Perron (2009), however, the MDF test of Harvey, Leybourne, and Taylor (2013) is invariant to the magnitude of any trend breaks. Such robustness enables the MDF test to incorporate breaks of different sizes and achieve good size and power properties.

Our analysis finds the nonlinearity in the temperature trend to be more complex than what has been reported in other studies. Allowing for a single trend shift may not be sufficient to uncover stationarity in surface temperature. Nonetheless, when two to three breaks in the level and/or the slope of the trend function are permitted, we can reject the stochastic trend hypothesis. Overall, our results augment and reinforce those of Estrada and Perron (2016) that both global and hemispheric temperatures exhibit nonlinear trend stationarity.

II. The data

This study examines yearly averaged data from two widely known instrumental temperature datasets. The first one is the HadCRUT v4.5.0.0 dataset, which contains data on global and hemispheric temperature anomalies with respect to the base period 1961–1990. It is compiled by the Hadley

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Center of the UK Met Office in collaboration with the Climatic Research Unit at the University of East Anglia. The data studied span the years from 1850 through 2015. The second one is a dataset developed by the National Oceanic and Atmospheric Administration (NOAA). The data analysed cover the period from 1880 to 2015 and are from the NOAA Global Surface Temperature (NOAAGlobalTemp) v4.0.1 dataset, which provides temperature anomaly data relative to the 1971–2000 reference period. This dataset, originated from a merged land-ocean surface temperature analysis (formerly known as MLOST), is made available by NOAA's National Centers for Environmental Information.

III. Conventional unit root test results

In the preliminary analysis, various efficient procedures are used to test for a stochastic trend in global and hemispheric temperature data. They include the GLS-detrended Dickey–Fuller (DF) test put forward by Elliott, Rothenberg, and Stock (1996). This test, referred hereafter as the $\mathrm{DF}^{\mathrm{GLS}}$ test, examines the null hypothesis of a unit root ($\rho = 1$) against local stationary alternatives of $\rho = 1 - \bar{c}/T$ for $\bar{c} > 0$ and sample size T. Let L be the standard lag operator. For a time series $\{y_t\}$, the $\mathrm{DF}^{\mathrm{GLS}}$ test is conducted based on the following autoregression:

$$(1 - L)\tilde{y}_t = \beta_0\tilde{y}_{t-1} + \sum_{j=1}^{p} \beta_j(1 - L)\tilde{y}_{t-j} + u_t \qquad (1)$$

where $\beta_j \text{for} j = 0, 1, \ldots, p$ are coefficient parameters, u_t is an error term and $\tilde{y}_t$ is defined by

$$\tilde{y}_t = y_t - \hat{\varphi}' z_t \qquad (2)$$

with $\hat{\varphi}$ being the estimated regression coefficient of $y_{\bar{\rho}} = (y_1, (1 - \bar{\rho}L)y_2, \ldots, (1 - \bar{\rho}L)y_T)'$ on $z_{\bar{\rho}} = (z_1, (1 - \bar{\rho}L)z_2, \ldots, (1 - \bar{\rho}L)z_T)'$ for $\bar{\rho} = 1 - \bar{c}/T$. In the case with a linear trend, $\bar{c} = 13.5$ and $z_t = (1, t)'$. The $\mathrm{DF}^{\mathrm{GLS}}$ test statistic is the t-ratio for testing $\beta_0 = 0$ against $\beta_0 < 0$.

Ng and Perron (2001) extends Perron and Ng's (1996) analysis and introduces an improved class of unit root tests called the $\overline{\mathrm{M}}^{\mathrm{GLS}}$ tests by incorporating GLS detrending. The $\overline{\mathrm{M}}^{\mathrm{GLS}}$ test statistics are given by

Table 1. Results from conventional unit root tests.

	HadCRUT			NOAAGlobalTemp		
Test	Global	NH	SH	Global	NH	SH
DF^{GLS}	-1.44	-1.80	-1.29	-1.75	-1.94	-0.93
$\overline{\mathrm{MZ}}_\alpha^{\mathrm{GLS}}$	-3.09	-5.27	-3.08	-3.44	-3.34	-2.47
$\overline{\mathrm{MZ}}_t^{\mathrm{GLS}}$	-0.89	-1.26	-1.02	-1.01	-0.93	-0.93
$\overline{\mathrm{MSB}}^{\mathrm{GLS}}$	0.29	0.24	0.33	0.29	0.28	0.38

Notes: Finite-sample critical values are obtained from 50,000 simulation replications based on $T = 150$. At the 5% level of significance, the critical values are computed to be -2.81 ($\mathrm{DF}^{\mathrm{GLS}}$), -15.77 ($\overline{\mathrm{MZ}}_\alpha^{\mathrm{GLS}}$), -2.77 ($\overline{\mathrm{MZ}}_t^{\mathrm{GLS}}$) and 0.18 ($\overline{\mathrm{MSB}}^{\mathrm{GLS}}$). None of the test statistics here is statistically significant.
NH: northern hemisphere; SH: southern hemisphere.

$$\overline{\mathrm{MZ}}_\alpha^{\mathrm{GLS}} = \left(T^{-1}\tilde{y}_T^2 - s_{AR}^2\right)\left(2T^{-2}\sum_{t=2}^{T}\tilde{y}_{t-1}^2\right)^{-1} \qquad (3)$$

$$\overline{\mathrm{MZ}}_t^{\mathrm{GLS}} = \left(T^{-1}\tilde{y}_T^2 - s_{AR}^2\right)\left(4s_{AR}^2 T^{-2}\sum_{t=2}^{T}\tilde{y}_{t-1}^2\right)^{-1/2} \qquad (4)$$

$$\overline{\mathrm{MSB}}^{\mathrm{GLS}} = \left(T^{-2}\sum_{t=2}^{T}\tilde{y}_{t-1}^2/s_{AR}^2\right)^{1/2} \qquad (5)$$

where $s_{AR}^2 = (T - p)^{-1}\sum_{t=p+1}^{T}\hat{u}_t^2/(1 - \sum_{j=1}^{p}\hat{\beta}_j)^2$ is an autoregressive spectral density estimator with $\hat{\beta}_j$ and $\hat{u}_t$ being the corresponding estimates of β_j and u_t in Equation 1.

Table 1 summarizes the unit root test results. All the tests performed include a linear trend and have the lag order p selected using the modified Akaike information criterion (MAIC) (Ng and Perron 2001) with a maximum order of 10 allowed. Despite using highly efficient tests, we remain unable to uncover stationarity in the global, northern hemisphere and southern hemisphere temperature series. In none of the cases can the stochastic trend hypothesis be rejected at the 5% significance level.

IV. Allowance for multiple trend breaks

It is generally known that the presence of structural change can seriously bias unit root tests towards under-rejection of the nonstationarity hypothesis. While a structural break represents an infrequent event, it can induce spurious persistence and create the appearance of permanent shocks, thereby confounding unit root tests and undermining their ability to detect stationarity. To account for the structural-break possibility, we apply the MDF test proposed by Harvey, Leybourne, and Taylor (2013). This test extends Perron and Rodriguez's (2003) analysis by allowing for multiple possible trend breaks at unknown times. The MDF test is based on the infimum of the sequence of GLS-based DF test statistics over all possible breakpoints within a selectively trimmed range.

Consider, in general, a process with up to m possible breaks. Let $\mathbb{I}(\cdot)$ be the indicator function and $\lfloor . \rfloor$ be the floor function. While $\mathbb{I}(\cdot) = 1$ when its argument is true and 0 otherwise, $\lfloor . \rfloor$ gives the integer part of its argument. Denote by $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_m)'$, a vector of unknown break fractions in an ascending order and by $\boldsymbol{\kappa} = (\tau_1 T, \ldots, \tau_m T)'$, their corresponding break dates. For $j \in \{1, \ldots, m\}$, the structural break occurring at time $t = \tau_j T$ is captured by $\mathrm{DU}_t^{\tau_j} = \mathbb{I}\left(t > \tau_j T\right)$ for a level shift or by $\mathrm{DT}_t^{\tau_j} = \left(t - \tau_j T\right)\mathbb{I}\left(t > \tau_j T\right)$ for a slope change in the trend function, where $\tau_j \in \Lambda = [\tau_L, \tau_U]$ with $0 < \tau_L < \tau_U = 1 - \tau_L < 1$. We set the trimming parameter $\tau_L = 0.10$.

Breaks may occur in either the level and/or slope of the trend function. Three alternative model specifications are entertained: Model A features a nonlinear trend with possible level shifts; Model B admits possible breaks in the trend slope and Model C permits both level and slope changes. Let $\mathrm{DU}_t^{\boldsymbol{\tau}} = (\mathrm{DU}_t^{\tau_1}, \ldots, \mathrm{DU}_t^{\tau_m})'$ and $\mathrm{DT}_t^{\boldsymbol{\tau}} = \left(\mathrm{DT}_t^{\tau_1}, \ldots, \mathrm{DT}_t^{\tau_m}\right)'$. Define $\mathbb{Z}_t = (1, t, \mathrm{DU}_t^{\boldsymbol{\tau}})'$ in Model A, $\mathbb{Z}_t = (1, t, \mathrm{DT}_t^{\boldsymbol{\tau}})'$ in Model B and $\mathbb{Z}_t = (1, t, \mathrm{DU}_t^{\boldsymbol{\tau}}, \mathrm{DT}_t^{\boldsymbol{\tau}})'$ in Model C. The GLS-detrended series $\breve{y}_t$ is then obtained as

$$\breve{y}_t = y_t - \hat{\phi}'\mathbb{Z}_t \tag{6}$$

Where $\hat{\phi}$ contains the parameter estimates from regressing $y_{\bar{\rho}} = (y_1, (1 - \bar{\rho}L)y_2, \ldots, (1 - \bar{\rho}L)y_T)'$ on $\mathbb{Z}_{\bar{\rho}} = (\mathbb{Z}_1, (1 - \bar{\rho}L)\mathbb{Z}_2, \ldots, (1 - \bar{\rho}L)\mathbb{Z}_T)'$ with $\bar{\rho} = 1 - \bar{c}/T$ for some $\bar{c} > 0$. For the noncentrality parameter, we use $\bar{c} = 17.6$ when $m = 1$, $\bar{c} = 21.5$ when $m = 2$, and $\bar{c} = 25.5$ when $m = 3$, as in Harvey, Leybourne, and Taylor (2013). Let $\mathrm{DF}_{\bar{c}}^{\mathrm{GLS}}(\boldsymbol{\tau})$ be the t-ratio for testing $\psi_0 = 0$ against $\psi_0 < 0$ from the autoregression in

$$(1 - L)\breve{y}_t = \psi_0 \breve{y}_{t-1} + \sum_{k=1}^{p} \psi_k (1 - L)\breve{y}_{t-k} + \varepsilon_t \tag{7}$$

Table 2. Results from the MDF test.

Test	HadCRUT			NOAAGlobalTemp		
	Global	NH	SH	Global	NH	SH
$\mathrm{MDF}_1^{\mathrm{GLS}}$ for one-break models ($m = 1$)						
Model A	-2.47	-2.40	-4.20	-2.21	-2.53	-2.25
Model B	-2.81	-3.32	-3.10	-2.91	-3.25	-3.74
Model C	-2.99	-3.30	-4.43	-3.00	-3.24	-3.92
$\mathrm{MDF}_2^{\mathrm{GLS}}$ for two-break models ($m = 2$)						
Model A	-4.17**	-3.04	-4.92**	-3.38	-3.17	-4.11**
Model B	-3.78	-3.62	-3.67	-3.96	-3.65	-4.66
Model C	-5.12**	-7.18**	-6.02**	-4.35	-5.87**	-4.82
$\mathrm{MDF}_3^{\mathrm{GLS}}$ for three-break models ($m = 3$)						
Model A	-4.51**	-4.35	-5.78**	-4.04	-4.08	-4.55**
Model B	-5.54**	-8.05**	-5.09	-5.03	-7.42**	-4.88
Model C	-8.61**	-8.16**	-6.85**	-7.38**	-7.58**	-5.76**

Notes: Finite-sample critical values are obtained using 30,000 simulation replications with $T = 150$. For the 5% significance level, the critical values are estimated to be -3.39 (Model A), -3.88 (Model B) and -3.98 (Model C) for $m = 1$; -3.89 (Model A), -4.74 (Model B) and -4.96 (Model C) for $m = 2$; -4.37 (Model A), -5.51 (Model B) and -5.68 (Model C) for $m = 3$. Statistical significance at the 5% level is indicated by double asterisks.

where $\psi_k, k = 0, 1, \ldots, p$ are coefficients to estimate and ε_t is an error term. The following infimum statistic can be applied to test for stationarity:

$$\mathrm{MDF}_m^{\mathrm{GLS}} = \inf_{\boldsymbol{\tau} \in \Phi_\eta} \mathrm{DF}_{\bar{c}}^{\mathrm{GLS}}(\boldsymbol{\tau}) \tag{8}$$

where $\Phi_\eta = \{(\tau_1, \ldots, \tau_m) : \tau_1, \ldots, \tau_m \in \Lambda$ and $\left|\tau_j - \tau_i\right| \geq \eta > 0 \ \forall\ i \neq j\}$. The parameter η determines the minimum separation of neighbouring breaks, and we use $\eta = 0.10$. The lag order p is, again, chosen using the MAIC. Since we should not include too many breaks than necessary to detect stationarity, the number of possible breaks is limited to no more than 3 (i.e. $1 \leq m \leq 3$).

Table 2 presents the MDF test results. In general, the results can vary depending on the number of possible breaks permitted. For models with a single break, there is no significant evidence to reject a stochastic trend. When two to three breaks are included instead, we find significant evidence rejecting the stochastic trend hypothesis. The form of breaks allowed may also matter. Compared to the other models considered, Model C – which admits both level and slope shifts – seems able to yield stronger and broader evidence against the stochastic trend hypothesis.

V. Conclusion

Both attribution and prediction of global warming hinge on proper modelling of the stochastic process of temperature changes. This study employs a state-of-the-art econometric method to examine whether the Earth's surface temperature contains a stochastic trend. We observe that structural change may have occurred more than once in the level and/or the slope of the trend function. With proper allowance for multiple trend breaks, we find significant evidence that both global and hemispheric temperatures are a stationary process. Our results add to and reinforce those of Estrada and Perron (2016). Global and hemispheric temperatures may appear to have a stochastic trend when in fact they are stationary around a nonlinear deterministic trend and structural breaks are responsible.

Disclosure statement

No potential conflict of interest was reported by the authors.

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