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Structural change and long-run reversion in the *ex ante* real interest rate

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This study examines recent US data on Treasury inflation-protected securities and presents new evidence supporting the Fisher hypothesis. The real interest rate may appear to display no mean reversion when in fact it is mean-reverting and a structural shift is responsible. According to break date estimates, the structural change occurred shortly after the collapse of Lehman Brothers at the height of the US financial crisis. The timing also coincided with the launch of quantitative easing by the Federal Reserve.

Keywords: real interest rate; Fisher effect; mean reversion; structural break; quantitative easing

JEL Classification: E43; G15

I. Introduction

A perennial issue in monetary economics concerns the empirical validity of the Fisher hypothesis, which posits an equilibrium relationship between the nominal interest rate and expected inflation. For the Fisher relationship to hold, the *ex ante* real interest rate – the difference between the nominal interest rate and expected inflation – should be mean-reverting. Many studies, however, find it empirically difficult to reject a unit root in real interest rates. If a unit root exists, the real interest rate would not exhibit mean reversion even in the long run, thus casting serious doubt on the Fisher hypothesis. The presence of a unit root would also be inconsistent with consumption-based asset pricing models (Rose, 1988; Lai, 2008).

When testing the Fisher effect, a complication arises because the *ex ante* real interest rate depends on expected inflation, which is not directly observable. The usual approach uses actual inflation as a proxy for expected inflation. The resulted analysis of the *ex post* real interest rate is, in effect, a joint test of the Fisher effect and rational expectations. An alternative approach uses some survey measure of inflation expectations (Lai, 2004; Sun and Phillips, 2004; Kaliva, 2008). The reliability and validity of survey data can be called in question, however. Inflation expectations can also be rather heterogeneous across individuals. It is far from clear how individual expectations should be combined or aggregated (Xiong and Yan, 2010). This study takes another different approach by using indexed bond data. A relatively new data-set on US Treasury inflation-protected securities (TIPS) is employed to test the Fisher effect. Inflation expectations are embedded in TIPS pricing. The yield spread between nominal Treasuries and TIPS provides a market-based measure of expected inflation over the maturity horizon.

The possible existence of structural change can also complicate empirical analysis of real interest rates (Evans and Lewis, 1995; Malliaropulos, 2000; Rapach and Wohar, 2005; Lai, 2008). While a structural break is an infrequent event, it can induce spurious persistence and create the appearance of permanent shocks, thereby confounding unit-root tests and undermining their ability to detect long-run reversion. The structural-break issue is particularly relevant to the US data under study, given the occurrence of such an unprecedented event as the launch of quantitative easing (QE) by the Federal Reserve amid a deepening financial crisis in 2008. With proper allowance for a mean shift, this study finds supportive evidence of the Fisher effect. Moreover, break date estimates from sequential tests indicate that the structural change likely took place during the peak of the financial crisis in late 2008.

II. The Data and Preliminary Test Results

Real constant maturity Treasury (CMT) rates of 5- and 7-year TIPS are examined in this study. These rates are derived from the US Treasury's real yield curve at fixed maturities based on market closing bid-side yields of actively traded Treasury securities. The real CMT rate data beginning from 2003 are made available by the Federal Reserve. Both end-of-week and end-of-month data are studied for the sample period from January 2003 through December 2014.

Since the usual augmented Dickey–Fuller (ADF) test is known to have low power to reject a unit root even when there is none, two additional tests with improved efficiency and power – called the DF-weighted symmetric (DF-WS) test and the DF-generalized least squares (DF-GLS) test – are applied. The increase in test power is achieved through using either WS or GLS estimation.

Let L be the lag operator and $\{y_t\}$ be a time-series process:

$$y_t = \rho y_{t-1} + \sum_{j=1}^{p} \beta_j (1-L) y_{t-j} + e_t \tag{1}$$

where β_j is a model coefficient and e_t is a random error.

The DF-WS test (Park and Fuller, 1995) involves minimizing a weighted sum of squared errors:

$$\begin{aligned}\Psi(\rho,\beta_1,\ldots,\beta_p) &= \sum_{t=p+2}^{T} w_t \left[y_t - \rho y_{t-1} - \sum_{j=1}^{p} \beta_j (1-L) y_{t-j} \right]^2 \\ &\quad + \sum_{t=p+2}^{T} (1-w_{t-p}) \left[y_{t-p-1} - \rho y_{t-p} \right. \\ &\qquad \left. - \sum_{j=1}^{p} \beta_j (1-L) y_{t-p-j} \right]^2 \end{aligned} \tag{2}$$

where the weighting scheme w_t is specified by

$$\begin{aligned} w_t &= 0 && \text{for } 1 \le t \le p+1 \\ &= (t-p-1)/(T-2p) && \text{for } p+1 < t \le T-p \\ &= 1 && \text{for } T-p < t \le T \end{aligned} \tag{3}$$

The DF-WS statistic, $\tau_{WS}(p)$, is computed as the t-statistic for testing $\rho = 1$ against $\rho < 1$.

The DF-GLS test (Elliott *et al.*, 1996) examines the unit-root null hypothesis ($\rho = 1$) against the local stationary alternative of $\bar{\rho} = 1 + \bar{a}/T$ with $\bar{a} < 0$. The test entails the following regression:

$$(1-L)\tilde{y}_t^{\mu} = \lambda \tilde{y}_{t-1}^{\mu} + \sum_{j=1}^{p} \theta_j (1-L) \tilde{y}_{t-j}^{\mu} + u_t \tag{4}$$

where u_t is an error term and $\tilde{y}_t^{\mu}$, the locally demeaned series under the local alternative, is given by

$$\tilde{y}_t^{\mu} = y_t - z_t \hat{\psi} \tag{5}$$

with z_t being a unit vector and $\hat{\psi}$ being the simple regression coefficient of y_t^L on z_t^L, for which $y_t^L = (y_1, (1-\bar{\rho}L)y_2, \ldots, (1-\bar{\rho}L)y_T)'$ and $y_t^L = (z_1, (1-\bar{\rho}L)z_2, \ldots, (1-\bar{\rho}L)z_T)'$ by setting $\bar{a} = -7$. The DF-GLS statistic, $\tau_{GLS}(p)$, is the t-ratio for testing $\lambda = 0$ against $\lambda < 0$.

Table 1 summarizes the unit-root test results. The lag order p is selected based alternately on the Schwarz information criterion (SIC) and the Akaike information criterion (AIC). Despite using highly efficient tests, we remain unable to uncover mean reversion in real interest rates. In none of the cases can the unit-root hypothesis be rejected.

Table 1. Results from conventional unit-root tests

	Monthly data (T = 144)		Weekly data (T = 626)	
	5-year rate	7-year rate	5-year rate	7-year rate
SIC lag selection:				
$\tau_{ADF}(p)$ statistic	−1.891	−1.517	−1.890	−1.887
Lag p	0	2	0	0
Critical value (α = 0.05)	−2.877	−2.867	−2.861	−2.861
$\tau_{GLS}(p)$ statistic	−1.577	−1.053	−1.260	−0.946
Lag p	0	2	0	0
Critical value (α = 0.05)	−2.078	−2.060	−1.980	−1.980
$\tau_{WS}(p)$ statistic	−1.811	−1.660	−1.942	−1.784
Lag p	2	2	0	0
Critical value (α = 0.05)	−2.586	−2.586	−2.551	−2.551
AIC lag selection:				
$\tau_{ADF}(p)$ statistic	−1.568	−1.517	−1.578	−1.584
Lag p	2	2	9	9
Critical value (α = 0.05)	−2.867	−2.867	−2.851	−2.851
$\tau_{GLS}(p)$ statistic	−1.291	−1.053	−1.131	−0.862
Lag p	2	2	9	9
Critical value (α = 0.05)	−2.060	−2.060	−1.965	−1.965
$\tau_{WS}(p)$ statistic	−1.729	−1.617	−1.828	−1.730
Lag p	4	4	9	9
Critical value (α = 0.05)	−2.593	−2.593	−2.559	−2.559

Notes: The sample statistics of the ADF, DF-GLS and DF-WS tests are given by $\tau_{ADF}(p)$, $\tau_{GLS}(p)$ and $\tau_{WS}(p)$, respectively. These are all one-sided lower tail tests. The lag parameter p is chosen alternately by the SIC and the AIC with a maximum lag of 12 allowed. Critical values for the ADF test are computed using response surface estimates (Cheung and Lai, 1995). For both the DF-GLS test and the DF-WS test, critical values are obtained through Monte Carlo simulations with 50 000 iterations. None of the test statistics reported in the table above is statistically significant.

These results fail to provide support for the Fisher hypothesis. We next explore the structural-break possibility.

III. A Robust Test for a Mean Shift

A partial sum (PS) procedure proposed by Vogelsang (1998) is applied to test for a mean shift in real interest rates. The PS test is attractive because it remains valid whether or not the time series is stationary. Consider a process with a mean shift at time, say, $t = k$:

$$y_t = \mu + \delta DU_t^k + \zeta_t \tag{6}$$

where μ and δ are models of coefficients, $DU_t^k = 1$ for $t \geq k$ and 0 otherwise, and ζ_t is a general zero-mean process. The mean-shift process can be rewritten in terms of partial sums:

$$s_t(y) = \mu t + \delta DT_t^k + \xi_t \tag{7}$$

where $s_t(y) = \sum_{j=1}^{t} y_j$, $\xi_t = \sum_{i=1}^{t} \zeta_i$ and $DT_t^k = (t-k)DU_t^k$. Let $\text{PS}_T(k)$ equal T^{-1} times the Wald statistic for testing $\delta = 0$. Without knowing the actual shift date, $\text{PS}_T(k)$ is computed over the entire range of possible dates, $\Lambda = \{t_m, t_m + 1, \ldots, T - t_m\}$, where t_m = Integer $(T/10)$. A mean-exponential PS statistic can be used to test for a mean shift at unknown time:

$$\begin{aligned}&\text{ExpPS}_T(b_\alpha)\\ &\quad = \ln\{T^{-1}\sum_{k\in\Lambda}\exp(\text{PS}_T(k)/2)\}\exp(-b_\alpha J_T^*)\end{aligned} \tag{8}$$

where b_α is a scaling parameter and $J_T^* = \inf_{k\in\Lambda} J_T(k)$ with $J_T(k)$ equal to T^{-1} times the Wald statistic for testing $c_1 = \ldots = c_q = 0$ in the following regression:

$$y_t = \mu + \delta DU_t^k + \sum_{j=1}^{q} c_j t^j + e_t \tag{9}$$

Based on optimality properties, $q = 9$ and b_α depends on the desired significance level (α) such that

$$\begin{aligned}b_\alpha = &-8.986 + 42.543\pi - 60.427\pi^2 + 29.432\pi^3\\ &+ \exp(-99.324 + 100\pi)\end{aligned} \tag{10}$$

where $\pi = 1 - \alpha$.

The results of the PS test are reported in Table 2. There is strong evidence of a significant mean shift in real interest rates. The null hypothesis of no mean shift can be soundly rejected in all the cases under study. Unit-root testing should thus include a structural shift under the alternative.

Table 2. Results from the mean-exponential PS test for a mean shift

	Monthly data ($T = 144$)		Weekly data ($T = 626$)	
	5-year rate	7-year rate	5-year rate	7-year rate
$\text{ExpPS}_T(b_\alpha)$ statistic	8.560**	14.816**	7.997**	13.965**
Critical value ($\alpha = 0.05$)	2.040	2.040	2.034	2.034

Notes: The PS test is a one-sided upper tail test. Its test statistic, $\text{ExpPS}_T(b_\alpha)$, contains a choice parameter that depends on the desired significance level, α, under optimality conditions (Vogelsang, 1998). Critical values are computed using the Monte Carlo method based on 50 000 simulation replications. Statistical significance at the 5% level is indicated by double asterisks (**).

IV. Unit-Root Testing with a Mean-Shift Alternative

The real interest rate dynamics are re-examined using sequential unit-root tests, which can permit a possible mean shift at an unknown date. Treating the break date as unknown is desirable since any arbitrarily fixed date can invite data-mining criticisms. The likely break date may still be estimated from the data with a sequential search procedure. Two approaches are available to model structural change. One is the additive outlier (AO) model that views the break as happening abruptly, and the other is the innovational outlier (IO) model that allows the break to occur gradually over a transition period (Banerjee *et al.*, 1992; Perron and Vogelsang, 1992; Zivot and Andrews, 1992; Vogelsang and Perron, 1998).

For the AO model, the dynamics of ζ_t in Equation 6 are considered to follow $(1 - \rho L)A(L)\zeta_t = B(L)v_t$, where $A(L)$ and $B(L)$ are lag polynomials with stable roots and v_t is white noise. A demeaned series $\tilde{y}_t^a$ is first constructed as follows:

$$y_t = \mu + \eta DU_t^k + \tilde{y}_t^a \tag{11}$$

where η represents model parameter and $\tilde{y}_t^a$ represents the LS residual series. Next, a test for $\alpha_{AO} = 0$ under the unit-root hypothesis is performed based on the following regression:

$$(1 - L)\tilde{y}_t^a = \sum_{j=0}^{p} \omega_j D_{t-j}(k) + \alpha_{AO}\tilde{y}_{t-1}^a + \sum_{j=1}^{p} \phi_j(1 - L)\tilde{y}_{t-j}^a + u_t \tag{12}$$

where ω is a model parameter, u_t is a random error, and $D_{t-j}(k) = 1$ for $t = k + j - 1$ and 0 otherwise. For given values of k and p, the t-statistic for testing $\alpha_{AO} = 0$ against $\alpha_{AO} < 1$ is denoted by $\tau_{DF}(\text{AO}, k, p)$. The estimated break date, k_B, is determined by minimizing $\tau_{DF}(\text{AO}, k, p)$ over $k \in \Lambda$. The unit-root test statistic is $\tau_{DF}(\text{AO}, k_B, p) = \inf_{k\in\Lambda} \tau_{DF}(\text{AO}, k, p)$.

The IO model entertains situations in which a structural break may happen gradually over time. The mean-shift process is specified as

$$y_t = \mu + \varphi(L)(\eta DU_t^k + v_t) \tag{13}$$

where v_t is the process innovation and $\varphi(L)$ is a polynomial function in lags, capturing the lingering effects of a gradual break. The unit-root test entails the following regression:

$$(1 - L)y_t = \mu + \omega D_t(k) + \eta DU_t^k + \alpha_{IO}y_{t-1} + \sum_{j=1}^{p} \psi_j(1 - L)y_{t-j} + \varepsilon_t \tag{14}$$

where ε_t is a random error term. Let $\tau_{DF}(\text{IO}, k, p)$ be the t-ratio for testing $\alpha_{IO} = 0$ against $\alpha_{IO} < 0$. The break point, k_B, is estimated by minimizing $\tau_{DF}(\text{IO}, k, p)$ over $k \in \Lambda$, yielding the unit-root test statistic $\tau_{DF}(\text{IO}, k_B, p) = \inf_{k\in\Lambda} \tau_{DF}(\text{IO}, k, p)$.

Table 3 contains the results from sequential unit-root tests. The results are not sensitive to the lag selection method. With a mean shift included under the alternative, the unit-root hypothesis can be rejected at $\alpha = 0.05$ in all but one of the cases when using the AO (abrupt change) model. Significant evidence against a unit root can also be found in all the cases when using the IO (gradual change) model. Evidently, the allowance for a structural shift helps uncover supportive evidence of long-run mean reversion in US real interest rates.

Furthermore, breakpoint estimates from the sequential search procedure suggest that the structural shift in US real interest rates happened in October or November of 2008, shortly after

Table 3. Results from sequential unit-root tests with a mean-shift alternative

	Monthly data ($T = 144$)		Weekly data ($T = 626$)	
	5-year rate	7-year rate	5-year rate	7-year rate
SIC lag selection:				
τ_{DF}(IO, k_B, p) statistic	−4.920**	−4.988**	−4.758**	−4.951**
Lag p	0	0	0	0
Critical value ($\alpha = 0.05$)	−4.391	−4.391	−4.361	−4.361
Break date estimate k_B	Nov. 2008	Oct. 2008	Nov. 2008	Nov. 2008
τ_{DF}(AO, k_B, p) statistic	−4.926**	−4.365	4.592**	−4.854**
Lag p	0	0	1	3
Critical value ($\alpha = 0.05$)	−4.417	−4.417	−4.368	−4.368
Break date estimate k_B	Nov. 2008	Nov. 2008	Nov. 2008	Nov. 2008
AIC lag selection:				
τ_{DF}(IO, k_B, p) statistic	−4.479**	−4.988**	−4.638**	−4.658**
Lag p	2	0	9	9
Critical value ($\alpha = 0.05$)	−4.476	−4.476	−4.384	−4.384
Break date estimate k_B	Nov. 2008	Oct. 2008	Nov. 2008	Nov. 2008
τ_{DF}(AO, k_B, p) statistic	−4.926**	−4.213	−4.695**	−4.830**
Lag p	0	2	9	7
Critical value ($\alpha = 0.05$)	−4.519	−4.519	−4.413	−4.413
Break date estimate k_B	Nov. 2008	Nov. 2008	Nov. 2008	Nov. 2008

Notes: One-sided lower tail tests are conducted to test the unit-root null hypothesis against the alternative hypothesis of no unit root but a mean shift. The lag parameter p is chosen alternately by the SIC and the AIC with a maximum lag of 12 allowed. The break date estimate indicates the estimated month and year during which a break occurred. Critical values are generated using the simulation method based on 50 000 iteration runs. Statistical significance at the 5% level is indicated by double asterisks (**).

the collapse of Lehman Brothers. The timing also correlated with the beginning of the Federal Reserve's QE policy.

V. Conclusion

This study examines recent US data on real CMT rates of TIPS and provides new supporting evidence for long-run reversion in the *ex ante* real interest rate. The finding accords with the Fisher hypothesis. Without allowing for any structural break, conventional unit-root tests commonly fail to detect mean reversion in the real interest rate. The real interest rate process is, however, found to have experienced a substantial structural break. When sequential unit-root tests permitting a mean shift are conducted, significant evidence rejecting unit-root dynamics can be obtained in support of mean reversion. Interestingly, the structural shift in the real interest rate took place soon after the collapse of Lehman Brothers at the height of the financial crisis in 2008. The timing also coincided with the start of the Federal Reserve's QE program.

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